Opening Career Systems: The Case of Simultaneous Vacancies

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Abstract

This paper analyses whether competing firms should open their career systems from an incentive perspective. Applicants for a vacant position compete in a Tullock contest. If career systems are open, workers within a firm may compete harder, thereby exerting higher efforts. However, under simultaneous vacancies at different firms, competition over a specific job may become softer for applicants and lead to a reduction in applicants' efforts. We analyze incentives when firms compete over workers, yet serve separate product markets, as well as when firms compete in the same product market. We find that firms will typically open their career systems, though risks of ending up in situations of low competitive pressure among applicants are pronounced in both market structures. When firms serve the same product market, softening competition among workers may even be used as a strategic tool for weakening a rival.

Key Words: contest; externalities; recruiting; wage policy.

JEL Classification: C72; J2; J3.

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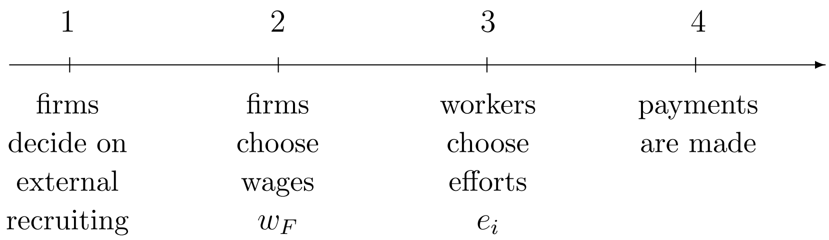
1 Introduction

Even in 2011, despite rather high rates of unemployment, many US industries saw a shortfall of skilled workers. Craig Giffi, vice chairman of the consulting form Deloitte, stated 'There's a tremendous shortage of skilled workers". According to a Deloitte survey, more than eighty percent of manufacturers lacked skilled production workers to a "moderate" or "severe" extent.[1] Especially in situations of few skilled applicants within their own hierarchies, firms may think about opening their vacant positions for external applicants as well. With competitive pressure from outside candidates, internal workers may become more motivated to exert high efforts to qualify for promotion. However, such effects may vanish if competing firms likewise search for skilled applicants and open their career systems. Competition for promotion may then become softer, thereby leading to low motivation in applicants. It is this interplay of incentives that our paper focuses on: Should competing firms open up their career systems or not? We analyze market structures in which firms that open their vacancies to external candidates compete over the same workers. We look at both, markets in which firms serve different product markets as well as markets in which firms also compete in terms of products. In both settings, we find that firms will tend to open their career systems, though chances of destroying overall competitive pressure are pronounced. If firms serve the same product market, we furthermore find that opening vacancies to external applicants may even be used as strategic tool for weakening a rival.

[1]The Wall Street Journal, Nov 26, 2011, Help Wanted: In Unexpected Twist, Some Skilled Jobs Go Begging,
http://online.wsj.com/news/articles/SB20001424052970203707504577010080035955166

2 The model

Our model is a variation of the model in Kräkel et al. (2014) which allows for simultaneous vacancies in both firms. There are two firms, A and B each employing one worker at its lower hierarchy level. Both firms intend to fill a vacancy at their higher hierarchy level. Let $t_F \in \{t_L, t_H\}$ be the talent of the worker that is employed by firm F at the lower hierarchy level $(F = A, B)$ while the other worker, being employed by firm $\hat{F}$, has talent $t_{\hat{F}} \in \{t_L, t_H\}$ $(\hat{F} \neq F)$. These talents are common knowledge. The timeline can be summarized as follows: First, both firms have to decide simultaneously on whether to accept an application from the external candidate or not. At the second stage, firms A and B attach wages w_A and w_B to their vacant positions. At the third stage, workers simultaneously choose efforts. Finally, workers are assigned to jobs and payments are made. We assume that each firm must fill its vacancy with one of the workers.



If a firm does not open its career system to the external worker, only the internal worker will compete for the vacant job. Career incentives will not work in such one-person contest since the internal candidate will be promoted with certainty. Consequently, he will exert zero effort. Firm F anticipates this behavior and chooses $w_F = 0$. As, by assumption, each firm must fill its vacancy with one of the workers, it is always optimal for the firms to accept external applications. However, they are free to choose between $w_F = 0$ and $w_F > 0$.

Given both firms' wages $w_F \geq 0$ and $w_{\hat{F}} \geq 0$, the two workers will compete for the higher wage $\max\{w_F, w_{\hat{F}}\}$. Let e_F denote the effort level chosen by the worker in firm F $(F = A, B)$ and $e_{\hat{F}}$ the effort of the worker being employed by firm $\hat{F}$. The worker of firm F gets $\max\{w_F, w_{\hat{F}}\}$ with probability $e_F/(e_F + e_{\hat{F}})$ and $\min\{w_F, w_{\hat{F}}\}$ with probability $1 - e_F/(e_F + e_{\hat{F}}) = e_{\hat{F}}/(e_F + e_{\hat{F}})$. He maximizes his expected utility

$$\max\{w_F, w_{\hat{F}}\} \frac{e_F}{e_F + e_{\hat{F}}} + \min\{w_F, w_{\hat{F}}\} \left(1 - \frac{e_F}{e_F + e_{\hat{F}}}\right) - \frac{e_F}{t_F}$$

$$= \min\{w_F, w_{\hat{F}}\} + \frac{e_F}{e_F + e_{\hat{F}}} |w_F - w_{\hat{F}}| - \frac{e_F}{t_F} \qquad (F, \hat{F} = A, B;\ F \neq \hat{F}).$$

A direct calculation shows that his optimal effort is given by

$$e_F^* = |w_F - w_{\hat{F}}| \cdot T_F \quad \text{with} \quad T_F := \frac{t_F^2 t_{\hat{F}}}{(t_F + t_{\hat{F}})^2}. \tag{1}$$

If both firms attach zero wages to their vacant positions or offer identical wages, both workers' optimal efforts will be zero.

3 Firms in Separate Product Markets

We first consider the case where both firms operate in different product markets. Firm F solves

$$\max_{w_F \geq 0} v(e_F^*) - w_F = \max_{w_F \geq 0} v(|w_F - w_{\hat{F}}| \cdot T_F) - w_F, \tag{2}$$

while at the same time firm $\hat{F}$ maximizes

$$v(|w_F - w_{\hat{F}}| \cdot T_{\hat{F}}) - w_{\hat{F}} \quad \text{with} \quad T_{\hat{F}} := \frac{t_F t_{\hat{F}}^2}{(t_F + t_{\hat{F}})^2}. \tag{3}$$

Assume for a moment that $w_{\hat{F}} = 0$. Then we have $e_F^* = w_F T_F$ according to (1). Hence, firm F's best response $w_F^* (w_{\hat{F}})$ to $w_{\hat{F}} = 0$ maximizes $v(w_F T_F) - w_F$:

$$w_F^*(0) = \begin{cases} \frac{1}{T_F} V\left(\frac{1}{T_F}\right) =: w_{F\hat{F}}^* & \text{if } T_F v'(0) > 1 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $w_{F\hat{F}}^*$ follows from F's first-order condition. Let analogously $w_{\hat{F}}^*(w_F)$ denote $\hat{F}$'s best response to w_F. Given $w_F = 0$, the best response $w_{\hat{F}}^*(0) = w_{\hat{F}F}^*$ if $T_{\hat{F}} v'(0) > 1$ can be derived in the same way as (4). Note that any relation $w_{LH}^* \gtreqless w_{HL}^*$ is possible since $\frac{(t_H+t_L)^2}{t_H t_L^2} > \frac{(t_H+t_L)^2}{t_H^2 t_L}$, but $V(\cdot)$ is monotonically decreasing. We obtain the following results for the optimal wage policies of firms F and $\hat{F}$ ($F, \hat{F} \in \{A, B\}; F \neq \hat{F}$):

Proposition 1 *Let* $\frac{t_H t_L^2}{(t_H+t_L)^2} v'(0) > 1$. *For simultaneous vacancies and firms operating in different product markets, there are two scenarios: (1) If workers are homogeneous (i.e.,* $t_F = t_{\hat{F}} =: t \in \{t_H, t_L\}$*), there are two pure equilibria* $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *and* $(w_F^*, w_{\hat{F}}^*) = (w_{F\hat{F}}^*, 0)$ *with* $w_{F\hat{F}}^* = w_{\hat{F}F}^* = \frac{4}{t} V\left(\frac{4}{t}\right)$. *There also exists a symmetric equilibrium in mixed strategies. (2) If workers are heterogeneous, so that* $t_F, t_{\hat{F}} \in \{t_H, t_L\}$, $t_F \neq t_{\hat{F}}$, *with* $w_{F\hat{F}}^* < w_{\hat{F}F}^*$, *then* $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *will be the unique equilibrium iff*

$$v\left(w_{\hat{F}F}^* T_{\hat{F}}\right) - v\left(w_{F\hat{F}}^* T_{\hat{F}}\right) > w_{F\hat{F}}^* + w_{\hat{F}F}^*; \tag{5}$$

otherwise there are two equilibria $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *and* $(w_F^*, w_{\hat{F}}^*) = (w_{F\hat{F}}^*, 0)$.

Proof. See Appendix A. ■

The condition given at the beginning of Proposition 1 excludes corner solutions where both firms choose zero wages. The results show that in the pure equilibria exactly one firm chooses a positive wage. This main finding is due to the fact that, given a zero wage w_F of firm F, the other firm $\hat{F}$

generates a positive externality by choosing a positive wage, which induces incentives to both workers. Firm F now must decide whether to free-ride and keep the zero wage $w_F = 0$, or to deviate to a strictly positive wage $w_F > 0$. However, in the latter case any rational positive wage must be at least twice as high as $w_{\hat{F}}$ because otherwise F destroys existing incentives (see (1)) at positive costs. The proof of Proposition 1 shows that such a deviation by F does not pay out for the firms in case of homogeneous workers or moderate degrees of heterogeneity.

If both workers are homogeneous or not too heterogeneous (i.e., $t_H - t_L$ is sufficiently small), then the two firms will face a coordination problem similar to the battle of the sexes. Both firms strictly favor the outcome that one of them creates incentives and the other one free rides by choosing a zero wage, but each of them prefers to be the free rider. If the firms fail to coordinate, they will end up in a situation with minimal (in the homogeneous case: zero) incentives. As worst possible outcome, both firms choose positive wages to generate incentives, but the two wages just offset each other in (1).

In case of strong heterogeneity, the two abilities t_H and t_L can differ so much that condition (5) is satisfied. Now workers' incentives are strictly more valuable to one of the two firms. This firm always prefers to generate incentives by choosing a positive wage irrespective of whether the other firm offers a positive wage or not. This strong preference solves the coordination problem. In the unique equilibrium, the first firm induces high incentives, whereas the latter firm optimally decides to free ride.

Numerical approximations show that mixed equilibria are also characterized by firms attempting to free-ride on the incentives set by the opponent. Figure 1 displays the equilibrium of a discretized game for a concrete choice of parameters.[2] In equilibrium, both firms set a wage of zero with a substantial probability and mix rather evenly over an interval above zero with the

[2]See Appendix B for technical details.

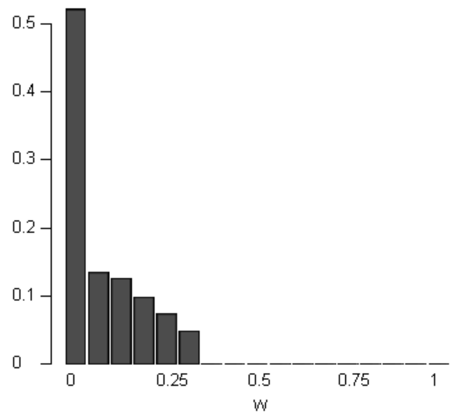


Figure 1: Symmetric mixed equilibrium in a discrete example

remaining mass.[3]

From a welfare perspective, the positive externality by inducing incentives for the external worker leads to an additional inefficiency. Consider, for example, the case of homogeneous workers ($t_F = t_{\hat{F}} = t$). Efficient or first-best effort e^{FB} maximizes $v(e) - \frac{e}{t}$, thus leading to

$$e^{FB} = V\left(\frac{1}{t}\right).$$

Optimal effort of a homogeneous worker in a two-person contest for a wage w is described by (1): $e^{**} = \frac{wt}{4}$. If a multi-plant corporation that consists of locations F and $\hat{F}$ organizes an internal job-promotion contest with wage w as winner prize, it will solve[4]

$$\max_{w \geq 0} 2v(e^{**}) - w = \max_{w \geq 0} 2v\left(\frac{wt}{4}\right) - w.$$

[3]We strongly conjecture that a similar mixed equilibrium exists also for the heterogeneous case. For discretized versions of the game, existence follows from results such as Harsanyi (1973) showing that typical games possess an odd number of Nash equilibria.

[4]Recall that the participation constraint is satisfied because of the limited-liability constraint and the worker's zero reservation value. As a direct implication, a firm prefers to choose a zero loser prize.

The solution is $w = \frac{4}{t}V\left(\frac{2}{t}\right)$ implying optimal effort

$$e^{**} = V\left(\frac{2}{t}\right),$$

which is strictly smaller than e^{FB} since V is monotonically decreasing. This inefficiency is well-known in the principal-agent literature: The firm cannot extract the full surplus from his workers, who are protected by limited liability (i.e., the firm is not allowed to choose a negative loser prize). Therefore, it induces less than efficient effort. In our context with a positive externality, the firm that sets a positive wage in equilibrium chooses $w^*_{F\hat{F}} = \frac{4}{t}V\left(\frac{4}{t}\right)$ according to Proposition 1. This wage leads to optimal effort

$$e^{*} = V\left(\frac{4}{t}\right) < e^{**} < e^{FB}.$$

The ranking of the three effort levels is quite intuitive. Since the value generated by the external worker does not accrue to firm F, optimal incentives are smaller than in the two-person job-promotion contest organized by the multi-plant corporation. Thus, from a welfare perspective both firms A and B should merge to a multi-plant firm in order to internalize the positive externalities in incentive creation.

4 Product Market Competition

Assume now that the two firms serve the same product market. Therefore, a firm F's profit function is described by

$$\psi\left(e_F - e_{\hat{F}}\right) - w. \tag{6}$$

Some additional assumptions are in order to make the firms' wage-setting game tractable. We assume that ψ has the following properties: ψ is a monotonically increasing, strictly positive, continuously differentiable and bounded function on $\mathbb{R}$ which is strictly concave on $\mathbb{R}^+$ and for which $\psi(x) + \psi(-x)$ is constant in x. The last assumption captures the idea that the two firms are competing for a market of fixed size.

Both firms now have a vacant position at the higher hierarchy level and simultaneously compete for the workers at the lower hierarchy levels. Let, w.l.o.g., $\Delta t := t_F - t_{\hat{F}} \geq 0$ with t_F and $t_{\hat{F}}$ denoting the talents of the two workers at the lower hierarchy level in firm F and firm $\hat{F}$, respectively. Hence, either both firms have equally talented workers in the initial situation or firm F has an H-type worker and firm $\hat{F}$ an L-type worker.

Equilibrium effort levels in the recruiting contest are again given by (1). Inserting into (6) shows that firm F solves

$$\max_{w_F \geq 0} \psi \left(|w_F - w_{\hat{F}}| \cdot \bar{T} \cdot \Delta t \right) - w_F, \qquad \text{with } \bar{T} := \frac{t_F t_{\hat{F}}}{(t_F + t_{\hat{F}})^2}, \tag{7}$$

whereas $\hat{F}$ solves

$$\max_{w_{\hat{F}} \geq 0} \psi \left(- |w_F - w_{\hat{F}}| \cdot \bar{T} \cdot \Delta t \right) - w_{\hat{F}}. \tag{8}$$

The solution of the game between firms F and $\hat{F}$ can be characterized as follows:

Proposition 2 *If workers are homogeneous (i.e.,* $\Delta t = 0$*), there exists the unique equilibrium* $(w_F^*, w_{\hat{F}}^*) = (0, 0)$*. Under heterogeneous workers (i.e.,* $t_F = t_H$ *and* $t_{\hat{F}} = t_L$*), either a pure equilibrium* $(w_F^*, w_{\hat{F}}^*) = (0, 0)$ *exists or an equilibrium in mixed strategies.*

Proof. See Appendix A. ■

If firms are homogeneous, no one can achieve a competitive advantage by inducing incentives. Consequently, each firm chooses a zero wage to save costs. If firms are heterogeneous but marginal returns are too small, there

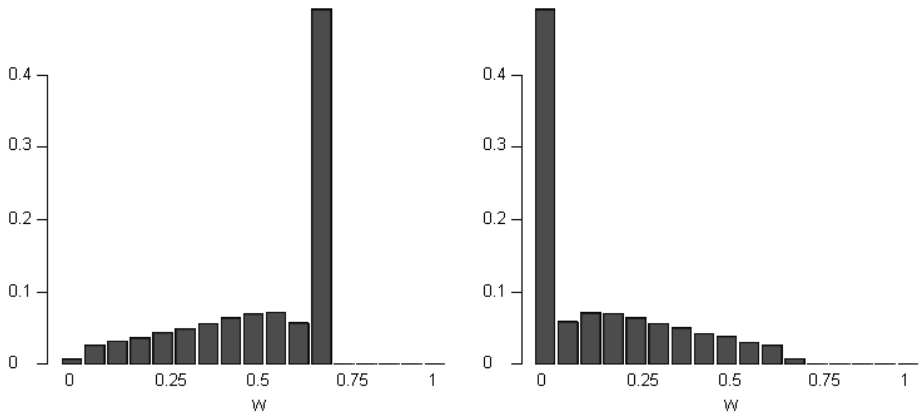


Figure 2: Mixed equilibrium strategies of F and $\hat{F}$ in a discrete example

will be a corner solution with both firms again setting zero wages. In case of heterogeneous firms and an interior solution, only mixed equilibria exist. Figure 2 displays a discrete approximation of such an equilibrium in a numerical example.[5] We see that firms mix over the same support. Firm F puts a substantial probability mass on the highest wage in the support, while firm $\hat{F}$ puts considerable mass on zero. In this example, firm F earns a payoff of about 2 while firm $\hat{F}$ earns about 0.5. If both firms would escape competition by setting a wage of zero, both would earn a bit more than 1.5. Compared to this, due to its stronger position firm F can gain about 0.5 while the sum of payoffs is reduced by 0.5 in equilibrium.

The logic behind this equilibrium is rather intricate: Firm F prefers the two wages to be as far apart as possible, while firm $\hat{F}$ prefers them to be close together. Moreover, both firms prefer to set small wages. Since firm F often plays high wages, firm $\hat{F}$ sometimes plays high wages as well in order to reduce the wage difference. But firm $\hat{F}$ cannot do this with too high probability because then firm F would have an incentive to free-ride and set

[5]See Appendix B for technical details.

a wage of zero – which has a direct negative effect on firm $\hat{F}$'s payoff. Thus, firm $\hat{F}$ sets a wage of zero with a substantial probability and firm F only attempts to free-ride with a comparatively small probability.

It is instructive to note that here firm $\hat{F}$ sets positive wages in order to *reduce* workers' incentives. This is because the firm knows that a strong wage difference between firms enhances competition between the two firms' workers. But since firm F has the more skilled workforce, this increased competition has the consequence that firm $\hat{F}$ loses market shares.

5 Conclusion

The analysis in our paper shows that opening vacancies to external candidates may be profitable in many market situations, even though losses due to competition between rival firms over candidates may be pronounced. We studied a market in which suitable applicants for a promotion are scarce, such that focusing on internal candidates alone will typically lead to low motivation in applicants. Out of this problem arises a natural incentive for firms to open their career systems. Future research should investigate situations in which competing firms try to fill vacancies, facing an internal workforce of medium suitability: From an incentives perspective, it seems intuitive that firms with very many well-suited internal candidates may focus on their own workforce when recruiting for a higher job position. Yet it is less clear how firms should optimally recruit with less internal candidates that may be eligible for promotion. All in all, the paper shows that the design of career systems can deeply influence market structures, and contribute to motivation in internal as well as external workforce.

Appendix A - Proofs

Proof of Proposition 1:

To prove the proposition, we can make use of the following two lemmas:

Lemma 1 *If $w_F > 0$, then $w^*_{\hat{F}}(w_F) \notin (0, 2w_F]$, $F, \hat{F} \in \{A, B\}$, $F \neq \hat{F}$.*

Proof. Given $w_F > 0$, firm $\hat{F}$'s objective function (3) is strictly larger for $w_{\hat{F}} = 0$ than for $w_{\hat{F}} \in (0, 2w_F]$. ■

Hence, investing in incentives can only be profitable to a firm if the existing incentives induced by the other firm are at least doubled. Otherwise, such investment would deteriorate existing incentives at positive costs.

Lemma 2 *If $w_F \geq w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$.*

Proof. $w^*_{\hat{F}}(w_F) \notin (0, 2w_F]$ due to Lemma 1. $w_{\hat{F}} > 2w_F$ cannot be a best reply to $w_F \geq w^*_{\hat{F}F}$ either: Problem

$$\max_{w_{\hat{F}}} v\left(\left(w_{\hat{F}} - w_F\right) T_{\hat{F}}\right) - w_{\hat{F}} \tag{9}$$

is solved by $w_{\hat{F}} = w^*_{\hat{F}F} + w_F \leq 2w_F$. Since (9) is strictly concave, $\hat{F}$ prefers $w_{\hat{F}} = 2w_F$ when choosing $w_{\hat{F}} \in [2w_F, \infty)$. However, $w_{\hat{F}} = 0$ would implement the same effort level at zero costs. ■

Lemma 2 states that a firm should completely save costs by choosing a zero wage if the other firm already induces sufficient incentives.

Now we can prove Proposition 1. We start with the case of *homogeneous workers*: $t_F = t_{\hat{F}} =: t$, so that

$$w^*_F(0) = w^*_{\hat{F}}(0) = \frac{4}{t} V\left(\frac{4}{t}\right) = \begin{cases} w^*_{HH} & \text{if} \quad t = t_H \\ w^*_{LL} & \text{if} \quad t = t_L \end{cases} \tag{10}$$

according to (4).

(1) If $w_F = 0$, then $w^*_{\hat{F}}(0) = w^*_{\hat{F}F} = w^*_{F\hat{F}} = \frac{4}{t}V\left(\frac{4}{t}\right)$ according to (10). Given this behavior of $\hat{F}$, firm F has no incentive to deviate (Lemma 2).

(2) If $w_F \geq w^*_{F\hat{F}} = w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$ by Lemma 2. Given $w_{\hat{F}} = 0$, firm F will choose $w^*_F(0) = w^*_{F\hat{F}}$ (see (10)) and no firm has an incentive to deviate.

(3) If $w_F \in (0, w^*_{F\hat{F}}] = (0, w^*_{\hat{F}F}]$, then $w^*_{\hat{F}}(w_F) \notin (0, 2w_F]$ (Lemma 1). There are three possibilities: (i) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \geq w^*_{\hat{F}F} = w^*_{F\hat{F}}$, then $w^*_F(w_{\hat{F}}) = 0$ (Lemma 2) and $w^*_{\hat{F}}(0) = w^*_{\hat{F}F} = w^*_{F\hat{F}}$ according to (10). (ii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} = 0$, then $w^*_F(0) = w^*_{F\hat{F}} = w^*_{\hat{F}F}$ (see (10)) and no one deviates. (iii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \in (2w_F, w^*_{\hat{F}F}] = (2w_F, w^*_{F\hat{F}}]$, then his best reply will solve

$$\max_{w_{\hat{F}} \in \left(2w_F, w^*_{\hat{F}F}\right]} v\left(\frac{(w_{\hat{F}} - w_F)\, t}{4}\right) - w_{\hat{F}}.$$

Since in case (iii), by assumption, $\hat{F}$ does not react by choosing zero effort (i.e., there is not a corner solution at zero as in case (ii))), the first-order condition can be applied, which leads to $w_{\hat{F}} - w_F = \frac{4}{t}V\left(\frac{4}{t}\right) = w^*_{\hat{F}F} \Leftrightarrow w_{\hat{F}} = w^*_{\hat{F}F} + w_F$. Because in case (iii) $\hat{F}$ is restricted to $w_{\hat{F}} \in (2w_F, w^*_{\hat{F}F}]$ and since the firm's objective function is strictly concave, $\hat{F}$ will choose the corner solution $w_{\hat{F}} = w^*_{\hat{F}F}$. Then $w^*_F(w_{\hat{F}}) = 0$ (Lemma 2) and no one deviates.

Existence of a symmetric mixed equilibrium can be shown as follows: Denote again by $w^*_{F\hat{F}}$ the wage one firm sets given that the other firm $\hat{F}$ sets a wage of zero. By the concavity of v, firm F will not respond with a wage strictly greater than $w^*_{F\hat{F}}$ to any strategy of firm $\hat{F}$ and vice-versa. Thus, any equilibrium of the restricted game where firms can set wages only in the interval $[0, w^*_{F\hat{F}}]$ must be an equilibrium of the original game as well. Payoffs in the restricted game are continuous and bounded and the action space is compact. Thus, by the main result of Becker and Damianov (2006) the restricted game possesses a symmetric equilibrium which is then also an equilibrium of the unrestricted game. Since there are no symmetric pure equilibria, this equilibrium must be in mixed strategies.

Second, we examine the *heterogeneous* case with $t_F, t_{\hat{F}} \in \{t_H, t_L\}$; $t_F \neq t_{\hat{F}}$. As any relation $w^*_{LH} \gtreqless w^*_{HL}$ is possible and the special case $w^*_{LH} = w^*_{HL}$ has already been discussed in the previous paragraph on homogeneity, without loss of generality it is sufficient to consider the remaining general case $w^*_{F\hat{F}} < w^*_{\hat{F}F}$ with $F, \hat{F} \in \{A, B\}$, $F \neq \hat{F}$.

(1a) If $w_F = 0$, then $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$. Given this behavior of $\hat{F}$, firm F has no incentive to deviate (Lemma 2).

(1b) If $w_{\hat{F}} = 0$, then $w^*_F(0) = w^*_{F\hat{F}}$. Given $w_F = w^*_{F\hat{F}}$, note that $w^*_{\hat{F}}(w^*_{F\hat{F}}) \notin (0, 2w^*_{F\hat{F}}]$. However, deviation to $w_{\hat{F}} \geq 2w^*_{F\hat{F}}$ can be optimal: Firm $\hat{F}$ solves

$$\max_{w_{\hat{F}} \geq 2w^*_{F\hat{F}}} v\left(\left(w_{\hat{F}} - w^*_{F\hat{F}}\right) T_{\hat{F}}\right) - w_{\hat{F}},$$

which leads to the solution $w_{\hat{F}} = w^*_{F\hat{F}} + w^*_{\hat{F}F} \geq 2w^*_{F\hat{F}}$. $\hat{F}$ will only deviate if this gives a higher expected profit compared to the initial situation $(w_F, w_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. This is not fulfilled if

$$v\left(w^*_{\hat{F}F} T_{\hat{F}}\right) - v\left(w^*_{F\hat{F}} T_{\hat{F}}\right) \leq w^*_{F\hat{F}} + w^*_{\hat{F}F}.$$

If this condition holds, $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$ is an equilibrium. Otherwise, $\hat{F}$ will deviate to $w_{\hat{F}} = w^*_{F\hat{F}} + w^*_{\hat{F}F}$ and we will end up in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ due to Lemma 2.

(2a) If $w_F \geq w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$ by Lemma 2. Given $w_{\hat{F}} = 0$, firm F will choose $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$.

(2b) If $w_{\hat{F}} \geq w^*_{F\hat{F}}$, then $w^*_F(w_{\hat{F}}) = 0$ by Lemma 2. Given $w_F = 0$, firm $\hat{F}$ will choose $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no firm has an incentive to deviate.

(3a) If $w_F \in (0, w^*_{\hat{F}F}]$, then $w^*_{\hat{F}}(w_F) \notin (0, 2w_F]$ (Lemma 1). There are three possibilities: (i) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \geq w^*_{F\hat{F}}$, then $w^*_F(w_{\hat{F}}) = 0$

(Lemma 2) and $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$. (ii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} = 0$, then $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. (iii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \in (2w_F, w^*_{F\hat{F}}]$, then his best reply will solve

$$\max_{w_{\hat{F}} \in \left(2w_F, w^*_{F\hat{F}}\right]} v\left(\left(w_{\hat{F}} - w_F\right) T_{\hat{F}}\right) - w_{\hat{F}}.$$

The first-order condition leads to $w_{\hat{F}} = w^*_{\hat{F}F} + w_F$. Because in case (iii) $\hat{F}$ is restricted to $w_{\hat{F}} \in (2w_F, w^*_{F\hat{F}}]$ and since the firm's objective function is strictly concave, $\hat{F}$ will choose the corner solution $w_{\hat{F}} = w^*_{F\hat{F}}$. Then $w^*_F(w_{\hat{F}}) = 0$ (Lemma 2) followed by $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no one further deviates.

(3b) If $w_{\hat{F}} \in (0, w^*_{F\hat{F}}]$, we also have to consider three possibilities: (i) If F reacts by choosing $w_F \geq w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$ followed by $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. (ii) If F reacts by choosing $w_F = 0$, then $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no one has an incentive to deviate. (iii) If F reacts by choosing $w_F \in (0, w^*_{\hat{F}F})$ we are back in the reasoning of (3a) resulting into $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$.

Proof of Proposition 2:
If workers are homogeneous, then $\Delta t = 0$ in (7) and (8), so that each firm chooses a zero wage as dominant strategy. The result on heterogeneous workers follows from the firms' best-response functions.

Lemma 3 *The best response of firm $\hat{F}$ satisfies $w^*_{\hat{F}}(w_F) \leq w_F$.*

Proof. The claim can be proved by contradiction. Suppose that $w_{\hat{F}} > w_F$ in (8). Then $\hat{F}$ strictly gains from switching to $w'_{\hat{F}}$ with $w'_{\hat{F}} < w_{\hat{F}}$ and $|w_F - w_{\hat{F}}| = |w_F - w'_{\hat{F}}|$. ■

If $\frac{t_H t_L \Delta t}{(t_H+t_L)^2}\psi'(0) < 1$, then F's best response to $w_{\hat{F}} = 0$ is given by $w_F^*(0) = 0$ and, applying Lemma 3, both firms end up in equilibrium $(w_F^*, w_{\hat{F}}^*) = (0,0)$. If

$$\frac{t_H t_L \Delta t}{(t_H+t_L)^2}\psi'(0) > 1, \tag{11}$$

then $(0,0)$ is not a Nash equilibrium and, as we argue next, no pure equilibrium exists. Note first that there cannot be a pure equilibrium where both firms set the same wage $w > 0$: Assume (w,w) is a Nash equilibrium. Since costs are linear, if neither firm wants to deviate to $\tilde{w} = w + z$ for some $z > 0$, then due to the linearity of costs both firms setting a wage of 0 must be a Nash equilibrium as well which contradicts our assumption that $(0,0)$ is not an equilibrium.[6]

Moreover, there cannot be a pure equilibrium where firm F sets w and firm $\hat{F}$ sets $\hat{w} > w$ by Lemma 3. Finally, we have to show that there cannot be an equilibrium where firms play wages w and $\hat{w}$ with $w > \hat{w}$. To simplify notation, we define $\phi(x) := \psi(x \cdot \bar{T} \cdot \Delta t)$. Since this is merely a rescaling, ϕ inherits all properties we assumed for ψ. In order to show that we do not have a Nash equilibrium, it suffices to consider small deviations which leave $w - \hat{w}$ positive. Therefore, we can leave away the absolute value and assume that firm F earns a payoff of $\phi(w - \hat{w}) - w$ and firm $\hat{F}$ earns a payoff of $\phi(\hat{w} - w) - \hat{w}$. By our assumption that $\phi(x) + \phi(-x)$ is constant, we have that $\phi'(x) = \phi'(-x)$ and $\phi''(x) = -\phi''(-x)$. Thus, if the first-order condition $\phi'(w - \hat{w}) = 1$ of firm F is satisfied, then $\hat{F}$'s first-order condition $\phi'(\hat{w} - w) = 1$ is satisfied as well. Now, consider second-order conditions: Since we assumed ϕ to be concave for positive arguments, we have $\phi''(w - \hat{w}) < 0$, so that firm F is indeed in a local maximum. The second derivative of firm $\hat{F}$'s payoff is $\phi''(\hat{w} - w) = -\phi''(w - \hat{w}) > 0$. Thus,

[6] Clearly, the opposite implication does not hold: From (w,w) firms can deviate to both, higher and lower wages while from $(0,0)$ they can only deviate to higher wages. Thus, $(0,0)$ being an equilibrium does not imply that (w,w) is an equilibrium as well.

firm $\hat{F}$ is in a local minimum and prefers to deviate to a marginally smaller or larger wage. Therefore, no pure strategy equilibrium exists.

It remains to be shown that a Nash equilibrium exists. Whenever no pure equilibrium exists this must be a mixed equilibrium. We first argue that firms do not play wages greater than $\overline{w} = \lim_{x\to\infty} \psi(x)$ in any equilibrium: Firm F can guarantee itself a non-negative payoff through setting a wage of 0 regardless of its opponent's strategy. Since setting a wage greater than $\overline{w}$ leads to a negative payoff for firm F regardless of the opponent's strategy, firm F does not play wages outside $[0, \overline{w}]$ in any equilibrium. Now, consider firm $\hat{F}$. By Lemma 3, if firm F plays a pure strategy w, firm $\hat{F}$ is better off playing w than setting a wage strictly higher than w. Likewise, if firm F plays a mixed strategy, firm $\hat{F}$ does not play wages above the support of F's strategy in equilibrium. Therefore, neither firm plays wages outside $[0, \overline{w}]$ in any equilibrium.

Thus, an equilibrium of the restricted game where firms can only set wages from $[0, \overline{w}]$ must be an equilibrium of the unrestricted game as well. In the restricted game, payoffs are bounded and continuous and the action space is compact. Therefore, we can apply the result of Glicksberg (1952) to show existence of equilibrium in the restricted game. This implies existence of equilibrium in the unrestricted game.

Appendix B - Details of Numerical Results

The numerical examples are based on the specification

$$v(x) = \psi(x) = \frac{\pi}{2} + \arcsin(12x)$$

which fulfills the requirements we made on v and ψ. In Section 2 we consider $t_F = t_{\hat{F}} = \frac{3}{2}$ while in Section 3 we choose $t_F = 3$ and $t_{\hat{F}} = 1$.[7] We discretized the game allowing only wages which are multiples of $\frac{1}{16}$. The discretized game was solved using the software package Gambit.[8]

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[7] The structure of the equilibria appeared to be robust to variations of these choices.

[8] Due to limitations of the software, finer discretizations were unavailable and payoffs had to be rounded to four valid digits. An even rougher discretization and rounding leads to results which are hardly distinguishable. Therefore, more accurate approximations can only be expected to lead to small quantitative corrections.