Information Asymmetry and Private Values in Second Price Auctions

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Abstract

I propose an auction model which reflects a situation in which one bidder faces competitors who are much better informed about the prize's quality. Situations like this might occur in market entry situations like the recent 5G spectrum auction in Germany, where after intense bidding, a new market entrant managed to obtain a significant share of the spectrum. I extend the standard independent private value model to capture this type of information asymmetry and retrieve unique equilibrium predictions in undominated strategies. In a sealed bid format, the uninformed bidder is at a clear disadvantage and can predominantly only succeed in the auction if the object's quality is low. An open auction format can completely level the playing field, implementing the first best efficient outcome.

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1 Introduction

Information plays a key role in auctions. It is of great importance for participants and auction designers alike which is why it has rightfully received a lot of attention in the literature. Uncertainty and lack of crucial information can give rise to all kinds of inefficiencies, for example by awarding the prize to an "undeserving" participant, i.e. one who does not value it highest, or by bankrupting the winner because the actual value of the prize was lower than the price he had to pay. Thus a social planner, who is solely interested in efficient allocation, must carefully take into account any information held by potential bidders and also information he might release himself (Schweizer and Szech, 2017). Participants of an auction, on the other hand, might go to great lengths to gain and exploit any information advantage they can get.

Designers of the German 5G spectrum auction in 2019 where surely aware of this when a potential market entrant applied for the auction. 1&1 Drillisch, subsidiary of US service provider United Internet, was looking to enter the market for the next generation of mobile internet providers, previously shared between three incumbents. One can imagine that 1&1 Drillisch saw itself in a peculiar situation going into the auction. Surely, the long-time incumbents had a far better understanding of the German mobile internet market and could therefore better predict the potential value of spectrum shares. Nonetheless, 1&1 Drillisch entered the auction determined, sending a clear signal to the incumbents by submitting relatively high initial bids (Reuters, 2019b). After intense bidding in 497 rounds over the course of 52 days, the auction raised 6.55 Billion Euros in total (Bundesnetzagentur, 2019). The achieved revenue was a lot higher than many experts expected (Financial Times, 2019) and there is reason to believe that the presence of a potential market entrant led to more aggressive bidding from the 3 incumbents, who tried to keep the status quo. In the end, 1&1 Drillisch was successful in the auction, acquiring 70 out of a total 420MHz of spectrum for 1.07 billion euros which saw their stock prices soaring in the aftermath (Reuters, 2019c).

The 2019 German 5G Auction was held as a multi-object auction, with 41 shares of the spectrum being auctioned simultaneously. The designers opted for an open ascending format which, according to my results, may have leveled the playing field and helped 1&1 Drillisch to enter the market. Yet, during the course of the auction and also immediately after, voices from Telecommunication providers spoke very negatively about the outcome, claiming the the high prices for spectrum represent a big setback in their development, with Vodafone Germany's CEO describing the outcome as "catastrophic" and market leader Deutsche Telekom's Germany chief Dirk Woessner stating "the auction leaves a bitter aftertaste" (Reuters, 2019a). So it is yet to be seen, if the new market entrant will be able to enjoy his prize.

I introduce a model which extends the standard private value model to capture the information disadvantage a market entrant might face. Bidders' valuations depend on their independent private signal and the object's quality (one can think of this as a state of the world). In contrast to previous approaches in the literature, the identity of the strongest bidder (i.e. the bidder with the highest valuation for the prize) depends on both the private signals *and* the state of the world. Information on the object's quality is extremely asymmetric. To one bidder, think of a market entrant, the object's quality and therefore his expected utility of obtaining it is unknown and he has to rely on a public signal. All other bidders, the incumbents, are perfectly informed about the object's quality and their valuation for the object. For simplicity, I focus on single object auctions but the dynamics of the equilibria can also give insights on multi-object formats.

I analyze this setup for the second price auction and for the strategically related English auction. This means that incumbents have a weakly dominant strategy, which lets me focus on the entrant's best response. These auction formats are particularly interesting, since the incumbents have only very limited ability to exploit their information advantage further. Nevertheless, I find that the entrant faces a considerable disadvantage in the sealed-bid SPA, where there is a high chance he will not compete for the high quality object, i.e. bidding in a way that he can only win if the object is of low quality. Also, two types of inefficiencies arise, including an ex-post Winners' Curse.

Previous work has shown that open auction formats can help less informed bidders (e.g. Compte and Jehiel (2007)), as they can infer valuable information from more informed bidders' behavior and this is exactly what I find. The English Auction erases the information disadvantage completely when there are at least two incumbents present, implementing an efficient allocation with certainty. This suggests that, from the perspective of a social planner, an open auction format is the superior choice when dealing with such a a market entry situation, negating the information asymmetries and securing an efficient outcome.

1.1 Related Literature

For decades, the independent private values model (IPV) has served as a work horse model when studying auctions, featured in virtually every book about auction theory and competitive bidding (e.g. Krishna (2009) or Klemperer (2004)). I extend the model by introducing different states of the world and drawing valuations independently for each state. Additionally, I vary bidders' information about which state of the world they're in.

In contrast to the private value models stands the common value approach. Historically this model was often used to study drilling rights auctions for oil and gas (Milgrom and Weber, 1982a). The assumption is that the value of the prize is the same for all bidders but every bidder has a different estimations of this value. These types of auctions have regularly led to the famous "Winner's Curse". Winning such an auction often means the bidder had the most optimistic estimation of the prize's value which was therefore likely to be lower than his estimate, resulting in over-payment. This phenomenon has been widely observed and replicated in the experimental literature (see e.g. Kagel and Levin (2002) for an extended survey on the Winner's Curse). The model in this paper also features a form of ex-post Winner's Curse as the uninformed bidder ends up overpaying for the prize with positive probability.

In the pure common value setting, information is key. Milgrom and Weber (1982a) show that if a bidder has strictly better information than another, the uninformed bidder's expected profit is zero in equilibrium. Wilson (1967) studies a related model where two bidders compete for a prize in a first price auction. One party knows the value with certainty while the other party does not. This way, the uninformed bidder inevitably has to randomize his bidding which leads to an equilibrium with quite different characteristics than the equilibria described in this paper. Also, the nature of the first price auction poses different incentives to the informed player, as he can choose how much profit he "shoots for". In this paper, the second price auction lets me simplify the informed bidders strategy and focus on the uninformed bidders best response.

One common approach to combine uncertainty about object's quality and private values is to separate every bidders valuation of the object into a common component, whose realization is unknown to all bidders, and a private component. For example, bidders valuation can be of the form $V_i = p_i v$ (or sometimes $V_i = v + p_i$) where p_i is a private signal and every bidder might have different estimates about the common component v (Wilson, 1992). A key implication of these models, is that the bidder with the highest private component has the highest valuation for the object, independent of the realization of the common component, meaning a social planner could allocate the object efficiently without knowing the value of v. In my model the efficient allocation (generally) depends on the objects quality. This situation might occur in the real world. One can imagine that in a spectrum auction, where future demand can be seen as a random variable, some firms might be the most effective provider when demand is relatively small, whereas there might be a different firm which is best equipped to scale up its operation if demand turns out to be very large.

This paper is also related to the literature of the "Market for Lemons", first introduced by Akerlof (1978), where it is shown that uncertainty about a product's quality can drive high quality products out of the market, leaving only "Lemons" (a term used for a sub-par used car) to be sold. This concept of adverse selection has been extended to auction theory. Lauermann and Wolinsky (2017), for example, show with a model where the auctioneer can invite bidders conditional on the object's quality, that competition softens and the auction fails to aggregate information resulting in a non-competitive price.

To my best knowledge, none of the existing models cover the type of value and information structure I propose nor contain it as a special case.

The rest of this paper is structured as follows: Section 2 describes the Model with all its assumptions, sections 3 and 4 present the equilibria of the sealed bid Second-Price Auction and the English Auction, respectively. A discussion of the results and its implications is provided in section 5 and section 6 concludes.

2 Model

An indivisible object or prize is sold in an auction. There are n bidders competing for the object. Let $q \in \{L, H\}$ denote the object's quality, which can be either low or high. Bidders have i.i.d. valuations for both types of the object, denoted v_L and v_H. These valuations are drawn according to continuous CDFs $F_L(v_L)$ and density functions $f_L(v_L)$ with support $[\underline{v_L}, \overline{v_L}]$, as well as $F_H(v_H)$ and $f_H(v_H)$ with support $[\underline{v_H}, \overline{v_H}]$, respectively. I assume that $\overline{v_L} \leq \underline{v_H}$ to ensure that the high quality object is always valued at least as high as the low quality object. I also assume that the density of the first-order statistic, denoted by $f^q_{1:n-1}(v_q)$ is continuous and well-defined for $q \in \{L, H\}$. The distribution functions as well as the information structure are common knowledge. Bidders are risk-neutral, so their utility for winning the auction is simply their respective valuation, depending on q, minus the price they have to pay.

$n-1$ bidders, the incumbents, are informed about the object's quality before the bidding phase, so they are certain of their valuation for the prize (one can also think of incumbents drawing only the relevant valuation after observing q). One bidder, the entrant (I will also use "she" to denote the entrant), only observes a public signal $p_H \in (0, 1)$, indicating the probability that the object is of high quality. p_H can also be seen as her prior belief that $q = H$.

I am interested in a full strategic analysis of the described game. The solution concept used is Bayes-Nash Equilibrium (BNE) in undominated strategies. A *strategy profile* consist of a strategy for each bidder. A strategy profile constitutes a BNE, if every strategy in that profile is a best response to all other strategies of that same profile, i.e. there is no strategy that yields a higher expected payoff given all other bidders' strategies.

All incumbents have a (weakly) dominant strategy in both the SPA and the English Auction, namely bidding truthfully, as shown in the seminal paper by Vickrey (1961). So in that sense, the informed bidders can not profitably alter their strategies to exploit their information advantage. I restrict my analysis to equlibria in which incumbents play this dominant strategy. The entrant has no such strategy, thus she must play a best response to the incumbents' strategies.

3 Sealed-bid Second Price Auction

This section provides the results of the sealed-bid second price auction (SPA), which is conducted as follows: First, all bidders draw their valuations and incumbents observe q. Then, each bidder privately submits a bid to the auctioneer (e.g. in a sealed envelope) who collects all bids and allocates the object to the highest bidder. The winner pays the value of the second highest bid, all other bidders pay nothing. Due to my assumptions, ties will occur with zero probability in equilibrium and are therefore not considered in my analysis.

I restrict my analysis to equilibria, in which incumbents play their weakly dominant strategy of bidding their (relevant) valuation. Let v_L and v_H be the entrant's valuation for the low and high quality object, respectively. Given the incumbents' strategy, bids in the interval $[0, v_L)$ for the entrant are strictly dominated (in expectation) by a bid of v_L. By bidding v_L she forfeits the chance to win if the object is of high quality (since she will be outbid by all informed bidders) but plays her optimal strategy if $q = L$. Similarly, bids in the interval $(v_L, \underline{v_H}]$ are dominated by v_L since they never win the object if $q = H$ and can never lead to a better outcome that v_L when the object is of low quality. Bids in the open intervals $(\underline{v_H}, v_H)$ and (v_H, ∞) are dominated by v_H: They lead to identical outcomes when the object is of low quality but v_H is strictly dominating in expectation when the object is of high quality.

The only undominated bids are therefore v_L and v_H. I define a *cutoff function* $G_n(v_L, v_H)$ as follows:

$$G_n(v_L, v_H) := p_H \int_{\underline{v_H}}^{v_H} f^H_{1:(n-1)}(v)(v_H - v)dv - (1 - p_H) \int_{v_L}^{\overline{v_L}} f^L_{1:(n-1)}(v)(v - v_L)dv$$

where $f^q_{1:(n-1)}(v) = (n-1)F^{n-2}(v)f(v)$ for $q \in \{L, H\}$ denotes the density of the first order statistic.

$G_n(v_L, v_H)$ represents a condition for the entrant's valuations. If the value of the function is greater than zero, her valuations are "high enough" and it is optimal for her to bid for the high quality object, i.e. submit a bid of v_H. If not, it is optimal for her to bid v_L, forfeiting her chance to win the auction if the object is of high quality but bidding optimally when it's of low quality.

Let $b : [\underline{v_L}, \overline{v_L}] \times [\underline{v_H}, \overline{v_H}] \to \mathbb{R}$ denote an entrant's bidding function.

Proposition 1. *The bidding function*

$$b(v_L, v_H) = \begin{cases} v_H & if \qquad G_n(v_L, v_H) > 0 \\ v_L & otherwise \end{cases}$$

for the entrant, together with the incumbents' dominant strategy of bidding truthfully constitute a BNE in undominated strategies.

For the formal proof, see appendix section A.1 which also shows that, apart from valuations where $G_n(v_L, v_H) = 0$, this $b(v_L, v_H)$ is the entrant's unique best response to the incumbents' dominant strategy.

So if the uninformed bidders' valuations are "high enough" and $G_n(v_L, v_H) > 0$ she will bid her high valuation, knowing that when the object is of low quality she will pay the incumbents' highest v_L, which might still yield a positive payoff. Otherwise she will bid her low valuation, forfeiting her chance to win the high quality object.

Lemma 2. *$b(v_L, v_H)$ is non-decreasing in both arguments. Additionally, $\forall v_L \in (\underline{v_L}, \overline{v_L}), v_H \in (\underline{v_H}, \overline{v_H})$:*

$$b(\overline{v_L}, v_H) = v_H$$
$$and$$
$$b(v_L, \underline{v_H}) = v_L.$$

Proof: The statements follow directly from the shape of $G_n(v_L, v_H)$ which is increasing in both arguments. To see this, assume that $v_H, v'_H \in [\underline{v_H}, \overline{v_H}]$ with $v'_H > v_H$. Then

$$\begin{aligned} G_n(v_L, v'_H) &= p_H \int_{\underline{v_H}}^{v'_H} f^H_{1:(n-1)}(v)(v'_H - v)dv - (1 - p_H) \int_{v_L}^{\overline{v_L}} f^L_{1:(n-1)}(v)(v - v_L)dv \\ &> G_n(v_L, v_H) + p_H \int_{v_H}^{v'_H} f^H_{1:(n-1)}(v)(v'_H - v)dv \\ &> G_n(v_L, v_H) \end{aligned}$$

This applies analogously to v_L.
When $v_L = \overline{v_L}$ the second term is zero and the first term is strictly positive as long as $v_H > \underline{v_H}$, yielding a strictly positive sum and therefore $b = v_H$. Similarly, when $v_H = \underline{v_H}$ the first term is zero and the second term is strictly negative if v_L is in the interior of its interval.
□

So both a higher valuation for the low quality object *and* the high quality object increase the probability of bidding the high value. Interestingly, a very high value for the high quality object can generally not ensure a bid of v_H, while a there is always a high enough v_L that does (also compare Figure 1).

Another thing to point out is that $G_n(v_L, v_H)$ does not depend on the absolute effect of the object's quality on bidders' valuations. In other words, the distance between the two intervals, given by $\underline{v_H} - \overline{v_L}$, does not affect the entrant's strategy. This is due to the fact that if she bids v_H and quality is low, she is guaranteed a prize in the lower interval $[\underline{v_L}, \overline{v_L}]$ (since all incumbents will place their bids there), thus rendering the distance between intervals irrelevant.

The condition ($G_n > 0$) that the entrant's valuation have to fulfill in order to bid for the high quality object strongly depends on n. Generally, the domain for which this condition holds shrinks rapidly when n increases (Compare Figure 1). However, the continuity of G_n together with the border case $G_n(\overline{v_L}, v_H) > 0$ (for all $v_H \neq \underline{v_H}$ and $N \geq 2$) ensures that the area of this domain is always greater than zero. So ex ante, there is always a positive probability that the entrant bids v_H and can win the high quality object.

In equilibrium, 2 types of inefficiencies occur with positive probability. Firstly, if $q = L$ and $b = v_H$, the uninformed bidder might win the object without having the highest valuation for it. In that case, she wins the objects at a price which is higher than her valuation, resulting in negative utility which can be seen as a form of the Winner's Curse. Secondly, if $q = H$ and $b = v_L$, the uninformed bidder might not win the object, even though she does have the highest valuation for it. The occurrence of these inefficiencies is generally not symmetric or equally likely, especially when n is larger, as the example in Figure 1 demonstrates: Even if the uninformed bidder has the highest

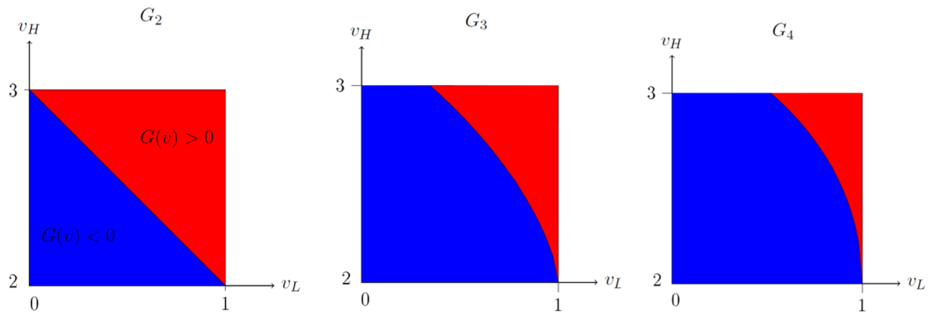

Figure 1: The domain of $G_n(v_L, v_H) > 0$ decreases in size when n increases. The entrant bids v_H when her values land in the red area and v_L in the blue area. ($v \in U[0,1] \times U[2,3]$).

possible valuation for the high quality object, there is a considerable chance she will still bid v_L when $n \geq 3$. This suggests, that the second type of inefficiency is generally more likely to arise (Compare also Table 1).

The auctioneer profits from the first type of inefficiency but loses out on potential revenue when the second type happens. The biggest impact in terms of revenue is in the two bidder case. Only there it is possible that the auctioneer sells the high quality object for a price in the lower interval $[\underline{v_L}, \overline{v_L}]$. In general, his expected revenue when compared to a situation with n informed bidders is higher when $q = L$ but lower when $q = H$, exactly because of these inefficiencies.

We will see later, that the open ascending auction eliminates both types of inefficiencies.

3.1 Example

As an example, let $p_H = \frac{1}{2}$ and valuations for the object are drawn uniformly from $[0, 1]$ and $[2, 3]$ for the low and high quality object, respectively. In the two bidder case with just one incumbent, the entrant's behavior in equilibrium simplifies to:

$$b = \begin{cases} v_H & if \qquad v_H + v_L > 3 \\ v_L & otherwise \end{cases}$$

In this simple case, the uninformed bidder bids high if the sum of her two valuations is higher than their expected value (See the left graph in Figure 1). Table 1 shows all possible outcomes of this auction, along with their ex ante probabilities of occurrence. So before drawing her valuations, the probability that the entrant will bid v_H is $\frac{1}{2}$. She also wins the auction with probability $\frac{1}{2}$, however with probability $\frac{1}{12}$ the price she has to pay is higher than her valuation. This is the ex post winner's curse. The other type of inefficiency also occurs with probability $\frac{1}{12}$, where the incumbent wins the high quality object without having the highest v_H.

Adding more incumbents leads to cutoff functions of the form:

$$G_n(v_L, v_H) = \frac{1}{2n} \cdot \Big((v_H - 2)^n - v_L^n + nv_L - (n-1) \Big)$$

The middle and right graph in Figure 1 illustrate the entrant's bidding function for $n = 3$ and 4, respectively. With 3 total bidders, the ex ante probability that the entrant bids v_H is only about 24.8%.

When $n = 4$ this number further decreases to around 15% and one can see that for every pair of v_L and v_H in the interior, there always exists a $N \in \mathbb{N}$ so that $G_n(v_L, v_H)$ is negative for all $n > N$. This demonstrates how the incumbent is forced out of the market for the high quality object as the number of incumbents gets large.

Table 1 further illustrates the occurrence of inefficiencies when increasing the number of incumbents[1]. While the total probability of an inefficient outcome is decreasing in the number of bidders, it does so asymmetrically. Since the entrant is more and more likely to settle for a chance of winning the low quality object, the probability of allocating the high quality object inefficiently is higher than the probability of mis-allocating the low quality object.

4 English Auction

In this section I present the result when applying the model to a standard open auction format. For this purpose, I model the English Auction in the

[1]Probabilities for $n \geq 3$ were calculated numerically with a maximum error of 10^{-5}. See appendix section A.2 for details

b	q	Entrant wins	Efficient Outcome	Probability (in %)		
				$N = 2$	$N = 3$	$N = 4$
v_H	H	Yes	Yes	16,67	7,00	3,84
		No	Yes	8,33	5,40	3,64
	L	Yes	Yes	16,67	8,24	4,98
		Yes	No	8,33	4,17	2,50
v_L	H	No	Yes	16,67	27,93	33,85
		No	No	8,33	9,67	8,66
	L	Yes	Yes	8,33	8,43	7,52
		No	Yes	16,67	29,17	35,00

Table 1: All Auction outcomes with corresponding (unconditional) probabilities of Example 1.

The column "Efficient Outcome" denotes the case where the object is awarded to the bidder with the highest valuation for it.

way of Milgrom and Weber (1982b), sometimes called "button auction" or Japanese auction: Each bidder presses a button at the start of the auction and keeps this button pressed as the price increases continuously. A bidder drops out of the auction by releasing his button, which can be observed by all bidders. The auction ends when there is only one bidder left pressing his button. This bidder wins and pays the price at which the second-to-last bidder released his button.

If we apply this auction format to our model, as long as there are at least two incumbents, the market entrant is no longer at a disadvantage (assuming incumbents play their dominating strategy). Imagine being the uninformed bidder in such a situation. Certainly you can keep your button pressed until the price reaches v_L. If at least one incumbent drops out before v_L you know the object is of low quality and dropping out at v_L is optimal. If v_L is reached without anyone dropping out, you simply stay in the auction. If an incumbent drops out before $\overline{v_L}$ is reached, you also know the object is of low quality and you should drop out immediately to not win the object at a price higher than your valuation. Generally this should always be possible, since there are at least 2 incumbents in the auction and the probability of them dropping out simultaneously is zero. If $\overline{v_L}$ is reached and all bidders are still pressing the button, you can be sure that the object is of high quality and stay in the auction until v_H is reached.

This strategy yields the efficient allocation with certainty. The entrant is no longer at risk of winning the low quality object at a price above his valuation and also always wins the object if he has the highest valuation for it. Although this does not necessarily translate into a higher expected revenue for the auctioneer, as he profits from one type of inefficiency in the sealed bid format but suffers from the other.

When there is only one incumbent in the auction, the result is effectively the same as in the sealed bid case: First, the market entrant stays in the auction until v_L is reached and drops out at that price if $G_2(v_L, v_H) < 0$. Otherwise he stays in the auction until v_H.

To see this, let's analyze the situation for the entrant when v_L is reached. At this point, there are 2 possible scenarios: Either $q = H$, or $q = L$ *and* the incumbent's valuation is higher than the entrant's. The entrant updates her belief that $q = H$, $p_H^{'}$ accordingly:

$$p_H^{'} = \frac{p_H}{p_H + (1 - p_H)(1 - F_L(v_L)}$$

Then she has to compare the utility from dropping out, which is zero, to the expected utility from staying in the auction, which is

$$(1 - p_H^{'}) \cdot E[v_L - \tilde{v}_L | \tilde{v}_L > v_L] + p_H^{'} \cdot F_H(v_H) \cdot E[v_H - \tilde{v}_H | \tilde{v}_H < v_H]$$

This expression is greater than zero if and only if $G_2(v_L, v_H) > 0$.[2]

5 Discussion

This sections provides comparisons with existing results and covers possible extensions to the model.

One of the motivating assumption about the model was that the identity

[2]When staying in the auction until v_H is more profitable than dropping out when the price is at v_L, it must clearly also be more profitably at any other point in $(v_L, \overline{v_L})$. Once $\overline{v_L}$ is reached, q can only be H and staying until v_H is optimal.

of the strongest bidder should depend on the object's quality. In this paper, this is reflected by the fact that the two valuations v_L and v_H are drawn independently, which is an assumption that might be seen as unrealistic. It is certainly more intuitive that a bidder with very high valuation for the high quality object would also tendencially have a high valuation for the low quality type and of course the model can be extended to include some correlation between the two values. I argue, however, that the results and observations drawn from this simple model with independent draws give a good indications of how equilibria with such extensions might look like.

Milgrom and Weber (1982a) showed, with very little additional assumptions, that in sealed-bid, pure common value auction, a less informed bidders expected profit is always zero. In other words, such a bidder is ex ante indifferent between attending the auction or staying out. In the model at hand, while she still faces a severe disadvantage, the uninformed bidders expected profit is always positive.

Another point I want to make is that the entrant's access to the high quality object is highly restricted even under risk-neutrality. Extending the model by adding risk-aversion, which has been of great interest as well in the literature (starting with works of Maskin and Riley (1984) and Matthews (1987)), leads to an even stronger restriction. Bidding for the high quality object always comes with the risk of overpayment if the object's quality is low, while the entrant can ensure a non-negative payoff (as well as strictly positive in expectation) by bidding his low valuation.

A natural extension of the model would be to increase the number of states of the world, i.e. $q \in \{q_1, q_2, ...\}$, with bidders drawing values for each q. One can easily show that, as long as the corresponding intervals remain disjoint, equilibria in this extended model have a similar structure. In equilibrium, the entrant's bid is exactly one of his drawn valuations. As the number of participants increases, the entrant is more likely to bid for the objects of lower quality.

What happens when there is more than one market entrant? Although I was not able to proof formally, I strongly suspect that the symmetric equilibrium has a similar structure, with the cutoff function being even more restrictive. One major difference with more entrants, is that the distance between intervals enters the equation since a high bid does no longer guarantee a price in the low interval if $q = L$. Nevertheless, since market entrants are a relatively rare occurrence in a lot of industries, I believe the model with just one entrant is still worth studying.

6 Conclusion

In this paper, I study an auction in an independent values setting in which one participant has less information about the object's quality than all the other participants. I characterize equilibria for the sealed-bid Second Price Auction and the English Auction. In the former, the uninformed bidder bids for the high quality object only if his private valuations are relatively high. As the number of participants increases, he restricts his attention more on the low quality object, essentially excluding him from the market for the high quality object. This leads to two types of inefficiencies: The uninformed bidder might win a low quality object at a price above his valuation or he might *not* win a high quality object even though his value for it is highest. In equilibrium, both inefficiencies arise with positive probability. An open ascending auction eliminates these inefficiencies at no cost, leading to an identical outcome as a setup where every bidder is perfectly informed and awarding the object to the bidder with the highest valuation for it. This suggests that open auction formats are superior in negating information asymmetries and therefore favor less informed bidders when compared to sealed-bid formats.

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A Appendix

A.1 Proof of Proposition 1

Since we already established that any bid other than v_L and v_H is dominated, it suffices to compare the expected revenue of the two potential bids, given that the informed bidders bid truthfully.
Bidding v_L yields

$$\begin{aligned} E[u(v_L)] &= (1-p_H)P\{v_L > v^L_{(1:n-1)}\} \cdot E[v_L - v^L_{(1:n-1)} | v_L > v^L_{(1:n-1)}] \\ &= (1-p_H)(F^L_{1:n-1}(v_L) \cdot \int_{\underline{v_L}}^{v_L} \frac{(v_L - v) f^L_{1:n-1}(v)}{F^L_{1:n-1}(v_L)} dv \\ &= (1-p_H) \int_{\underline{v_L}}^{v_L} f^L_{1:n-1}(v)(v_L - v) dv \end{aligned}$$

while a bid of v_H gives us

$$\begin{aligned} E[u(v_H)] = {} & (1-p_H)\, E[v_L - v^L_{(1:n-1)}] + \\ & + p_H \left(P\{v_H > v^H_{(1:n-1)}\} \cdot E[v_H - v^H_{(1:n-1)} | v_H > v^H_{(1:n-1)}]\right. \\ & = (1-p_H) \int_{\underline{v_L}}^{\overline{v_L}} f^L_{1:n-1}(v)(v_L - v) dv + p_H \int_{\underline{v_H}}^{v_H} f^H_{1:(n-1)}(v)(v_H - v) dv \end{aligned}$$

From this I can calculate when bidding v_H is more profitable than a bid of v_L

$$\begin{aligned} E[u(v_H)] \geq E[u(v_L)] \quad & \iff \\ p_H \int_{\underline{v_H}}^{v_H} f^H_{1:(n-1)}(v)(v_H - v) dv \geq (1-p_H) \int_{v_L}^{\overline{v_L}} f^L_{1:n-1}(v)(v - v_L) dv \quad & \iff \\ G_n(v_H, v_L) \geq 0 \quad & \end{aligned}$$

$\square$

When $G_n(v_L, v_H) = 0$ is the entrant indifferent between bidding v_L or v_H. So only bidding functions that map $b(v_L, v_H)$ to v_L when $G_n(v_L, v_H) < 0$, to v_H when $G_n(v_L, v_H) > 0$, and to either of those two values when $G_n(v_L, v_H) = 0$ are best response functions to the incumbents' strategy.

A.2 Calculations for Table 1

To efficiently calculate the respective probabilities in Table 1, I first perform a simple transformation. Remember that the cutoff function has the form

$$G_n(v_L, v_H) = \frac{1}{2n} \cdot \Big((v_H - 2)^n - v_L^n + nv_L - (n-1)\Big)$$

when $v_L \sim U[0,1]$, $v_H \sim U[2,3]$ and $p_H = \frac{1}{2}$.
So the function $f_n(v_L) = (v_L - nv_L + (n-1))^{\frac{1}{n}}$ maps all $v_L \in [0,1]$ to a $v'_H := f(v_L)$ with $G(v_L, v'_H + 2) = 0$. Using this function I can calculate

$$P(\text{Entrant bids}\, v_L) = \int_0^1 min(f_n(x), 1)dx$$

which is just the size of the blue areas in Figure 1 relative to the total area. I can further define the conditional distribution functions of v_L and v_H, given that the entrant bids v_L or v_H:

$$\begin{aligned}
F_L(v_L|b = v_L) &= \int_0^{v_L} \frac{min(f_n(x), 1)}{P(\text{Entrant bids}\, v_L)}dx \\
F_L(v_L|b = v_H) &= \int_0^{v_L} \frac{max(1 - f_n(x), 0)}{P(\text{Entrant bids}\, v_H)}dx \\
F_H(v_H|b = v_L) &= \int_0^{1} \frac{min(v_H, f_n(x))}{P(\text{Entrant bids}\, v_L)}dx \\
F_H(v_H|b = v_H) &= \int_0^{1} \frac{max(v_H - f_n(x), 0)}{P(\text{Entrant bids}\, v_H)}dx
\end{aligned}$$

With these conditional distribution functions I can now calculate the probabilities, with which the entrant's valuation is higher or lower than the highest incumbent's, given her own bid. Let $q_1, q_2 \in \{L, H\}$. Then

$$P(v_{q_1} < v^{q_1}_{(1:n-1)}|b = v_{q_2}) = \int_0^1 (n-1)x^{n-2} F_{q_1}(v_{q_1}|b = v_{q_2})dx$$

where $v^{q_1}_{(1:n-1)}$ denotes the highest of the $n-1$ incumbents' valuations for object with quality q_1 and the term $(n-1)x^{n-2}$ refers to the density of the first order statistic of the incumbents' valuations.

The entries in the table are then calculated by simply multiplying the respective probabilities, e.g. the probability that the entrant bids v_H, the object's quality is H *and* the entrant has the highest valuation for the object is: $P(\text{Entrant bids}\, v_H) \cdot \frac{1}{2} \cdot P(v_H > v^H_{(1:n-1)} | b = v_H)$