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Optimal disclosure of costly information packages in auctions☆

Nora Szech

Economic Theory II, University of Bonn, Lennéstr. 37, 53113 Bonn, Germany

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ABSTRACT

In an independent, private values, second-price auction with entry fees we discuss the way in which a seller should optimally spread costly information among the bidders. We find that marginal gross revenues do not generally behave monotonically in total information release. In the two bidder case, essentially, any asymmetric allocation of information dominates the symmetric information allocation. Even the bidder who gets less information is willing to pay a higher entry fee for asymmetric information allocations than for the symmetric one. His entry fee coincides with that of the better informed bidder. Losses from allocating an amount of information non-optimally can be substantial.

1. Introduction

Consider different firms competing in a takeover auction. They are all interested in the target, but not for the same reasons: Bidder A is interested in the clients list of the target. Bidder B is more focused on possible synergies to reduce production costs. Firm C is in need of the target's know-how. Most likely, it is not easy to get information on any of these aspects. Should the target be willing to open its books, show the client list, give the bidders access to its production processes? It is reasonable to assume that informing the bidders causes the target firm some costs—at least, revealing information takes costly time. Furthermore, it is likely that every bidder asks specific questions and has to be monitored while searching through the target. This leads to the main question: How much information should be given to the different bidders in order to maximize the auction's revenue if revealing information is costly?

To consider questions of this type we need a model that allows for giving out different amounts of information to different bidders. We consider the following setting: Each aspect a bidder is interested in about the target is wrapped in an information package. The seller (e.g. the target itself) possesses all these information packages. Yet giving out packages is costly, and so the seller will usually not give out all packages. In case if a bidder does not get one of his desired information packages, he sticks to his commonly known prior about this aspect of the target. The seller does not know what the information in the packages means to the bidders. He only takes into account that giving out more packages means that bidders will be better informed in the auction (so that they bid higher with some probability).

We assume that the seller can sell his information by charging ex ante entry fees from the bidders. Therefore, the seller can extract all expected surplus on the side of the bidders. Accordingly, the seller's problem is equivalent to maximizing social welfare and all our results on optimal allocations also apply to the problem of welfare maximization.

We find that with two bidders, allocating packages symmetrically is dominated by most asymmetric allocations. For example, concentrating all packages at one bidder always generates a weakly higher payoff than splitting up the same amount of information equally. For the n bidder case, such a general statement is not true. Yet we will also give examples where the restriction to symmetric information allocations leads to substantial reductions in the seller's revenue for the n bidder case.

With two bidders, both bidders are willing to pay the same maximum fee for the release of more packages—no matter how asymmetrically these packages will be allocated. Thus a bidder who is still uninformed is willing to pay a higher fee for additional information given to the other bidder than for receiving this amount of information himself.

Our study is related to some recent work on information acquisition and mechanism design. In particular, there are some other papers dealing with independent private values auctions where the seller chooses the degree to which he informs the bidders: Bergemann and Pesendorfer (2007) consider the case of no entry fees and no information costs. Esö and Szentes (2007)

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E-mail address: nszech@uni-bonn.de.

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allow for entry fees, but rule out information costs. Ganuza and Penalva (2010) rule out entry fees, but allow for information costs. Our study addresses the fourth case where entry fees can be charged and information costs are present. Esö and Szentes show an extraction-of-surplus result which is more general than ours as it allows for preliminary information on the side of the bidders. As informing the bidders is not costly to their seller, he will give out all the information he has. This is in contrast to our framework, where the seller has to face the trade-off between additional rents due to information revelation and additional costs. Hoffmann and Inderst (2009) generalize the results of Esö and Szentes (2007) to a setting where giving out information is costly. Their analysis is restricted to the case where there is only one bidder. In Ganuza and Penalva, the seller is restricted to inform the bidders symmetrically. Thus, our main question of how to allocate information optimally cannot be addressed in their framework. Bergemann and Pesendorfer (2007) consider a wide class of information structures and find that the information structure in the revenue-maximizing mechanism treats bidders asymmetrically. Hagedorn (2009) further explores the optimality of asymmetric over symmetric information structures in the same setting.

Szech (2010) also discusses the question of how to allocate information among bidders but in a simpler model of information release. In that setting, it holds that the seller optimally allocates information as asymmetrically as possible under weak conditions. The asymmetry results of the present study are stronger in the sense that the present model explicitly allows for effects such as cancelation between good and bad news through the release of additional information. It is weaker in the sense that only suboptimality of symmetric information policies holds. Thus the two papers complement each other.

There is also a literature on auction environments with costly information acquisition by the bidders. This includes Persico (2000), Bergemann and Välimäki (2002), Compte and Jehiel (2007), Bergemann et al. (2009), Shi (2011) and Crémer et al. (2009). Crémer et al. (2009) is closest to our study: They also consider a revenue-maximizing seller who can charge entry fees in an independent private values model and prove a full-extraction of expected surplus result.

All the papers which assume that better information can be acquired at a higher cost face the problem of how to model information which is, both, more valuable and more costly. The simplest approach, taken, e.g., by Compte and Jehiel (2007), Vagstad (2007), Crémer et al. (2007); Crémer et al. (2009), and Bergemann et al. (2009) is to assume that each bidder either stays completely uninformed or learns his valuation perfectly. In Hoffmann and Inderst (2009) and Szech (2010), the seller chooses probabilities with which each bidder learns his valuation perfectly. Other papers utilize more sophisticated models to allow for partial release of information: Persico (2000) and Bergemann and Välimäki (2002) order the cost of informative signals according to the signal's effectiveness (also known as accuracy), a concept which goes back to Lehmann (1988). Ganuza and Penalva (2010) and Shi (2011) introduce new classes of orders to rank the informativeness of signals based on different stochastic orders.

Our information package model of Section 3 takes an intermediate approach, allowing for a partial release of information but staying comparably simple and thus tractable: Each bidder's valuation is assumed to be the sum of m i.i.d. random variables (m packages of information). The seller decides how many packages he wants to reveal to each bidder at costs which depend on the total number of packages released. As discussed at the end of Section 3, this approach allows us not only to have an ordinal ranking (as in stochastic order approaches), but also to have a cardinal measure of how much information the bidders get.

Note that the comparative simplicity of our model does not necessarily imply that it is a less accurate description of the decision problem which, e.g., the seller of a house might face: Is such a seller really able to let bidder 1 learn whether his valuation for the house is above or below 50,000 $ (and nothing else) and bidder 2 whether his valuation is above or below 200,000 $? Yet this is only a small fraction of the power which the seller possesses, e.g., in the setting of Bergemann and Pesendorfer (2007). Thinking of such practical examples it becomes natural to study the decision problem of a seller whose power does not go beyond an imperfect control of the amount of information transmitted to each bidder.

The paper is organized as follows: Section 2 introduces the model and analyzes which entry fees the bidders are maximally willing to pay depending on how much information is given out. Section 3 – the main part which is also technically the most interesting section of this study – focuses on the two bidder case where information is spread over several packages. Section 4 briefly discusses the case of more than two bidders. Section 5 concludes. All proofs, including the calculations behind the examples, are in Appendix.

2. The model

A seller sells one indivisible object for which his valuation is zero via a second-price auction. There are n risk-neutral bidders with independent (but not necessarily identically distributed) valuations $X_1, \ldots, X_n$ with expected values $\mu_1, \ldots, \mu_n$.[1] The bidders do not know their valuations initially. The seller offers against entry fees to give to each bidder i a certain amount of information (represented by a σ-algebra $\mathcal{F}_i$) such that each bidder can calculate an estimate $\widetilde{X}_i = E[X_i|\mathcal{F}_i]$ of his valuation. Each bidder only receives information on his own type, but not on the other bidders' valuations. Let $\widetilde{X}^{(1)}$ and $\widetilde{X}^{(2)}$ be the two highest order statistics among these estimates.

The precise timing of the model is as follows: First, the seller announces individual entry fees to each bidder. He commits to giving out an information structure (describing which bidder will get how much information) and commits also to excluding all bidders that refuse to pay. Second, the bidders decide if they want to pay their fee. Third, the bidders who have paid get their information. Fourth, all bidders who have paid participate in the second-price auction. Without loss of generality, ties in the auction are broken with equal probability.

Throughout the paper, we assume that the bidders stick to their weakly dominant strategy of bidding the best estimate they have of their valuations. We assume that giving information to the bidders is costly to the seller. We will, however, not specify this assumption before the next sections when we further restrict the $\mathcal{F}_i$. In Proposition 1, we calculate the entry fees which the seller can maximally charge such that bidders still participate.

Proposition 1. *If the seller offers to release the information sets $(\mathcal{F}_1, \ldots, \mathcal{F}_n)$, each bidder i is willing to pay an entry fee of*

$$e_i = E[(X_i - \widetilde{X}^{(2)})1_{\{i \text{ wins}\}}] = E[(\widetilde{X}^{(1)} - \widetilde{X}^{(2)})1_{\{i \text{ wins}\}}].$$

This leads to a gross expected revenue for the seller of $E[\widetilde{X}^{(1)}]$.

[1] For convenience, it is assumed throughout the paper that all random variables are not almost surely constant. Without this assumption, all arguments still go through but some strict inequalities hold only weakly. We also assume that all random variables are L^1 integrable, i.e. $E[|\cdot|] < \infty$.

Thus the seller can extract all surplus from releasing more information, and this is an optimal selling mechanism in this framework. Note that, by Jensen's inequality and the convexity of the maximum, it follows that $E[X^{(1)}] \geq E[\widetilde{X}^{(1)}]$. Therefore, in the absence of costs, the seller prefers to release as much information as possible such that the bidders learn the realizations of their valuations X_i exactly.

With only two bidders, both bidders' entry fees coincide if they have the same prior estimates of their valuations $\mu_1 = \mu_2$:

Proposition 2. *Consider the setting of the previous proposition but with $n = 2$. Assume $\mu_1 \geq \mu_2$. Then bidder 1 pays*

$$e_1 = \frac{1}{2}E[|\widetilde{X}_1 - \widetilde{X}_2|] + \frac{1}{2}(\mu_1 - \mu_2)$$

and bidder 2 pays

$$e_2 = \frac{1}{2}E[|\widetilde{X}_1 - \widetilde{X}_2|] - \frac{1}{2}(\mu_1 - \mu_2).$$

Thus we see that the difference in entry fees between the two bidders always equals the difference in priors $\mu_1 - \mu_2$. This does not change even if one bidder gets much more information than his competitor. The reason is that more information can raise a bidder's valuation; but it can lower it as well, thus reducing the competition in favor of the other bidder: With two bidders, good news to the first bidder translate one-to-one into bad news to the second bidder. With more than two bidders, such a simple relation does not hold anymore.

Accordingly, in the setting with two bidders and a welfare-maximizing seller who does not set entry fees, the proposition shows that both bidders earn the same expected payoff, no matter how asymmetrically information is allocated.

3. The two bidder case

In this section, we consider a more concrete model of information release which is rich enough to capture the following decision of the seller: The seller may inform bidders asymmetrically although he could spread the same amount of information evenly as well. We will focus on the two bidder case and illustrate what changes with more bidders in the next section.

We now assume two bidders who have valuations $X_1 + \cdots + X_m$ and $Y_1 + \cdots + Y_m$ where $X_1, \ldots, X_m, Y_1, \ldots, Y_m$, the packages of information, are i.i.d. random variables with distribution function F and mean μ. Each package represents an independent privately-valued aspect of the object for sale. The revenue-maximizing seller decides how many packages each bidder should get. The seller has a cost of c per revealed package.

From Propositions 1 and 2, we know that if for some $k, j \leq m$ the seller reveals $X_1, \ldots, X_k$ and $Y_1, \ldots, Y_j$, his expected gross revenue is

$$E[\max(X_1 + \cdots + X_k + (m-k)\mu, Y_1 + \cdots + Y_j + (m-j)\mu)] \quad (1)$$

and each bidder pays an entry fee of

$$e_1 = e_2 = \frac{1}{2}E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_j + (j-k)\mu|]. \quad (2)$$

Then Proposition 3 shows that the seller prefers to give a fixed number of packages to one bidder, leaving the other bidder uninformed. Splitting up the packages evenly among the two bidders would give him lower revenues:

Proposition 3. (1) *Assume $2k \leq m$ packages are to be allocated by the seller. If the distribution of the packages is asymmetric around the mean, it is strictly more profitable to concentrate the $2k$ packages of information at one bidder than to split them equally between the two bidders.*

(2) *Assume $j \leq 2m$ packages are to be allocated by the seller. If the distribution of packages is symmetric around the mean, the seller's revenue does not depend on how the packages are allocated.*

Note that the proposition implies that a bidder may pay a higher fee for an additional package given to the other bidder than for receiving this information package himself.

The proof of Proposition 3 relies on the fact that for i.i.d. random variables X and Y

$$E[|X + Y|] \geq E[|X - Y|] \quad (3)$$

with equality if and only if the distribution of X and Y is symmetric around the mean. This result is found, for instance, in Jagers et al. (1995). It is already plausible from (2), the formula for the entry fees, that such a formula is useful for the proof: Shifting packages from bidder 1 to bidder 2 translates into turning plus signs into minus signs in the formula. Note that in the two bidder case, maximizing the difference between the higher and the lower order statistic (which is twice the entry fee) is equivalent to maximizing the higher order statistic.

Exploiting (3) a little more, we can generalize Part 1 of Proposition 3 as follows:

Proposition 4. *Assume $2k < 2m$ packages are to be allocated and the distributions are asymmetric. Then any split-up of the type $(2l, 2h)$ where $l > h$ and $l + h = k$ leads to a higher revenue for the seller than (k, k).*

To illustrate the proposition, consider the following example: Assume the seller wants to give out six packages in total. Then he should not give three packages to each bidder, but rather give four or six packages to one bidder. However, the proposition does not make a statement about giving five packages to one bidder and one package to the other bidder. Despite this obstacle (which does not seem to be easy to remove except in the case where each package can be rewritten as a sum of two i.i.d. random variables[2]), Proposition 4 says that splitting up information evenly is, essentially, the least profitable decision the seller can take.

Two effects drive the result: First, concentrating packages at one bidder may lead to a very high interim valuation of that bidder (e.g. if all packages he receives contain good news). Such a high interim valuation can never occur if the seller instead splits the total amount of packages evenly among the bidders. Second, giving all packages to one bidder makes sure that the other bidder, who receives no package, has an interim valuation of $m\mu$. Thus the first order statistic of the interim valuations, which our seller wants to maximize, cannot get smaller than $m\mu$. This insurance effect also has some power if a bidder receives not none, but few packages, and becomes less important when more bidders take part in the auction (as it is then unlikely that the first order statistic becomes small). Yet with few bidders, the insurance aspect of leaving bidders completely or nearly uninformed plays a crucial role.

We have seen that the splitting up of information equally is definitely not optimal. But which information policy is revenue-maximizing? Is concentrating all information at one bidder optimal in this sense? The answer can depend sensitively on the distribution F as we can see in the following example:

[2] This is possible for instance for infinitely divisible probability distributions like the exponential distribution.

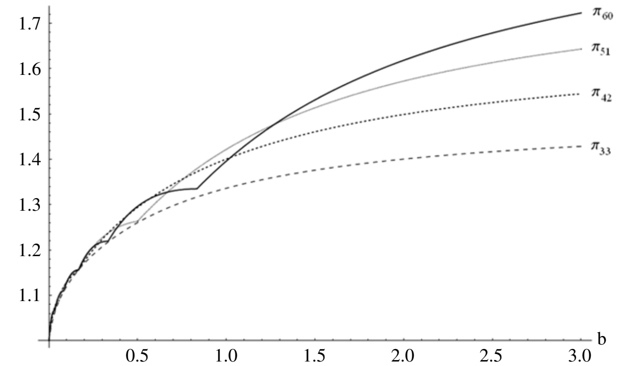

Fig. 1. Payoffs from different allocations in Example 1.

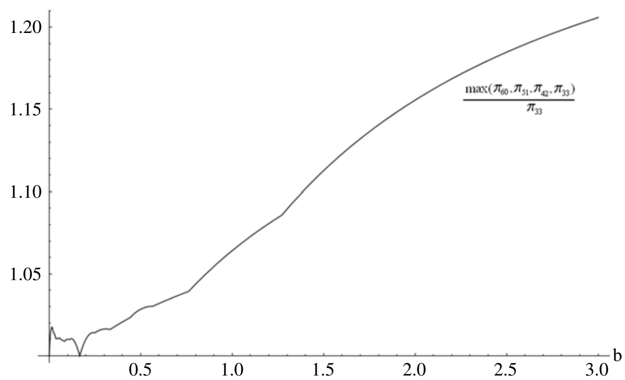

Fig. 2. Maximal loss in Example 1.

Example 1. Consider the question of how to allocate six packages of information optimally among two bidders whose valuations consist of six packages each. Assume that the packages take only two values, 0 and $\frac{1}{6}+b$, and have mean $\mu = \frac{1}{6}$. Via b, we vary the asymmetry of the probability distribution of the packages. An uninformed bidder's expected valuation is $6 \cdot \frac{1}{6} = 1$. Denote by π_{ij} the seller's expected gross revenue from giving i packages to one bidder and j packages to the other bidder. In Fig. 1, we see $\pi_{60}, \pi_{51}, \pi_{42}$ and π_{33} as functions of b. Recalling Propositions 3 and 4, it is not surprising that $\pi_{33}(b)$ is strictly dominated by the other curves (except for the symmetric case $b = \frac{1}{6}$ where all the four curves coincide). Beyond these facts, however, the seller's optimization problem is rather complex: The set of values of b for which concentrating all information at one bidder is optimal, consists of five disjoint intervals. Every asymmetric allocation is strictly optimal for some values of b. Hence it depends sensitively on b whether allocations (4, 2), (5, 1), or (6, 0) are revenue-maximizing.

Fig. 2 depicts $\max(\pi_{60}, \pi_{51}, \pi_{42})/\pi_{33}$, the relation between revenues from allocating six packages optimally and from allocating them equally. As b increases, the loss quickly reaches a substantial amount. For large b, allocating information optimally generates over 30% more revenue than allocating information equally.

Thus we see that the optimal allocation depends quite sensitively on the probability distribution of the packages.

So far we have only discussed how to allocate a given number of packages optimally. Let us now look at how many packages in total the seller should release. The following lemma shows that for some cost levels the seller will decide to release some, but not all information.[3] The next Lemma shows that the first package that is given out is the most profitable one:

Lemma 1. *The first package of information given out leads to a strictly higher increase in the seller's revenue than any additional package.*

The question arises how much additional expected revenue can be made by giving out a second, a third, or a fourth package. Is there a result such that the second package leads to a higher revenue increase than the third, the third package to a higher increase than the fourth, and so on? The following examples show that this depends on the probability distribution F:

Example 2. Assume that each bidder's valuation consists of two packages, i.e. $m = 2$.

(1) Assume that the packages are distributed uniformly on [0, 1]. Then the release of each further package leads to a smaller increase in the seller's expected revenue than the package released before. Hence we find concavity of expected gross revenue in the number of released packages.

(2) Assume that each package takes the values 0 and 1 with equal probability. Then the release of the second or the fourth package does not influence the seller's expected gross revenue. Thus, depending on the level of costs, the seller will give out no, one, or three packages of information. Notably, the seller will only inform the second bidder perfectly if $c = 0$.

(3) Assume that the packages are distributed exponentially with parameter 1. Then the first package leads to a higher increase in revenue than the second which again leads to a higher increase than the fourth package. The third package, however, leads to a smaller increase in revenue than the fourth. Thus, depending on the costs, the seller will release none, one, two, or four information packages: If the second bidder gets informed at all, he gets fully informed.

The latter two examples have shown that the sequence of the seller's expected gross revenues if he releases a total of l packages is generally not concave in l.[4] Yet the following lemma shows that the sequence is not too non-concave either, namely, that it is increasing and bounded from above by a concave function:

Lemma 2. *Let the bidders' valuations consist of m independent, identically distributed packages, each package with mean μ and standard deviation σ.*[5] *The seller's expected gross revenue if he releases a total of $l \le 2m$ packages is weakly increasing in l and bounded from above by $m\mu + \frac{\sigma}{2}\sqrt{l}$.*

Setting this upper bound in relation to the payoff $m\mu$ from giving out no information also gives us a rough upper bound on the maximal losses from allocating information suboptimally.

To close this section, we want to discuss two assumptions we made—first, that packages are identically distributed, and, second, that there is no preliminary information on the side of the bidders.

To discuss the first assumption, let us consider the situation where packages are not identically distributed. The proofs of our results on how to allocate information do not go through in that setting as we cannot rely on inequality (3). It is also intuitive that

3 This justifies the approach we have taken in this section so far: The lemma ensures that it is worthwhile to think about how an intermediate, fixed amount of packages should be split up among the bidders. Considerations of this kind were pointless if the seller always decided to give out all the packages he has.

4 Such nonconcavities, in a different context, are the main focus of Radner and Stiglitz (1984), see also Chade and Schlee (2002).

5 As we need a finite standard deviation for this lemma, we have to assume here (and only here) that the random variables are L^2 and not just L^1.

the results themselves do not carry over in general: Assume that X_1 and Y_1 had a much larger variance than the remaining packages. Then the seller should release X_1 and Y_1 first, even though this would be an equal split-up of information, in order to create as much variability in the interim valuations as possible.

The following proposition underlines – for the case of only one package per bidder – how the seller's allocation problem depends on the variability of the packages:

Proposition 5. *Consider the case $m = 1$, i.e., bidders have valuations X_1 and Y_1. Assume that X_1 and Y_1 are independent and have means μ_X and μ_Y. Then informing bidder 1 is strictly more profitable in expectation than informing bidder 2, if*

$$E[|X_1 - \mu_X|] > E[|Y_1 - \mu_Y|].$$

In the special case where $\mu_X = \mu_Y = \mu$, this condition becomes

$$E[|X_1 - \mu|] > E[|Y_1 - \mu|],$$

i.e., the bidder with the higher mean absolute deviation should be informed.[6] This condition is also equivalent to

$$E[(X_1 - \mu)^+] > E[(Y_1 - \mu)^+]. \tag{4}$$

As the uninformed bidder's valuation is certain and guarantees μ, the seller compares in (4) a call-option on the valuation of bidder 1 to a call-option on the valuation of bidder 2 (both with strike μ).

Our model of identically distributed, equally costly packages may look rather limited at first sight. This may be especially true if one identifies our information packages with concrete properties of the object for sale. Our packages should be taken as an abstract division of a large amount of information into small pieces which are only loosely related to concrete aspects of the object.[7] The package units provide an exact measure of how much information a bidder gets, and help us to compare the amounts of information different bidders receive not only in an ordinal, but also in a cardinal ranking. This is an advantage compared to approaches based on stochastic orderings as in Persico (2000) and Ganuza and Penalva (2010): Stochastic orderings can express that one bidder is better informed than another. Yet they do not specify what it means that, e.g., bidder 1 gets twice as much information as bidder 2. Our package model is a natural and relatively tractable way to achieve this goal.

Generally, the analysis of a model with non-identically distributed packages would be complicated by the same technical difficulties that made obtaining "clean" solutions difficult in the identically distributed packages model: Expectations of absolute values of sums and differences of random variables are much more difficult to handle than, for example, variances of sums and differences of random variables. Note, however, that the bounds of Lemma 2 immediately translate to any non-identically distributed, independent packages (with the sum of package variances instead of $m\sigma^2$).

Furthermore, via continuity arguments it should be possible to extend our analysis to the case of packages which are almost identically distributed. To illustrate this point, consider the case of two bidders with valuations $X_1 + X_2$ and $Y_1 + Y_2$. The X_i and Y_i are independent, X_1 and Y_1 are exponentially distributed with parameter 1, and X_2 and Y_2 are distributed exponentially with parameter λ. Assume that the seller releases two packages in total. An elementary calculation shows that for $0.97 < \lambda < 1.04$ it is optimal to inform one bidder fully. Hence we see that the optimality of concentrating information is robust around the value $\lambda = 1$ (at which all packages are identically distributed).

A second assumption of our analysis which may seem rather strong is that bidders do not hold preliminary private information. While the effects present in our analysis should still play a role in such a setting, the seller's allocation problem would typically be dominated by other concerns: He should sell information to bidders which value it highly while still keeping the auction sufficiently competitive. The bidders' willingness to pay for information depends crucially on their private information. This leads to a quite complex mechanism design problem of which a solution is beyond the scope of our study. So far, two important special cases of the problem have been considered in the literature: Esö and Szentes (2007) study the problem in the case where releasing information is costless (such that the seller always gives out all information). Hoffmann and Inderst (2009) study the one bidder case with costs of information. In both cases, the question of how to split up intermediate amounts of information among different bidders does not arise. Still, even for these "simple" cases the optimal mechanisms are intricate. Thus, finding the revenue maximizing mechanism in a model that allows for preliminary information and for unequal split-ups of information seems highly challenging.

Nevertheless, our analysis can solve the following non-trivial problem with preliminary information: An efficiency maximizing auctioneer decides about giving out costly information before the auction takes place. He does not charge fees for information provision. There are two bidders with valuations X_1+X_2 and Y_1+Y_2 where the X_i and Y_i are independent and identically distributed with an asymmetric distribution. Initially, bidder 1 privately knows X_1 while bidder 2 is uninformed. Assume that the seller wants to reveal one package in total. Then from Proposition 3 we can deduce the following: Revealing X_2, i.e., informing bidder 1 completely, strictly dominates revealing Y_1. Hence we can immediately see with the techniques we developed so far that the auctioneer unlevels the playing field further in a situation where he could level it as well.

Another case which is covered by our analysis is the one where bidders' valuations have some common value component which is commonly known and where the seller has in hand only the information which makes up the difference between the bidders:

4. More than two bidders

So far, we have focused on the two bidder case and found that a fixed amount of information will generally be allocated unequally among the bidders. To get a more complete picture we now have a brief look at examples with more than two bidders. The first example shows that we cannot hope for an equally general asymmetry result as in the two bidder case. It is a three bidder version of Example 1.

Example 3. Consider the problem of how to allocate six packages of information among three bidders. Assume that the probability distribution of the packages is the same as in Example 1. Let $\pi_{ijk}(b)$ denote the expected gross revenue from allocating i, j, and k packages to the three bidders. Fig. 3 compares the allocations π_{ijk}. See that $\pi_{222}(b)$ is no longer the universal worst choice for all b, but it is still far from optimal. The optimal information policy again depends sensitively on b.

In Fig. 3, the curves no longer coincide for the symmetric case $b = \frac{1}{6}$. To see why, we compare the three bidder case with the two bidder case: With two bidders, for any symmetric

6 Note that a random variable X having a larger absolute deviation than Y is not equivalent to X having a larger variance than Y. In many natural examples the two properties however go together.

7 One could, e.g., assume that $X_1 + \cdots + X_l$ stands for one aspect of the object, and $X_{l+1} + \cdots + X_m$ for another one. Alternatively, one could interpret the packages as hours the seller spends on informing the different bidders.

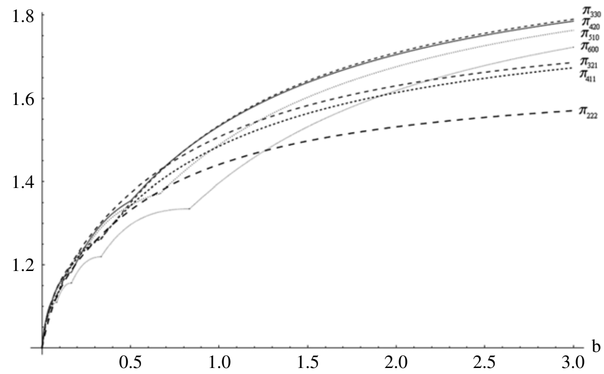

Fig. 3. Payoffs from different allocations in Example 3.

probability distribution, revenues are independent of the package allocation, e.g., $\pi_{20}\left(\frac{1}{6}\right) = \pi_{11}\left(\frac{1}{6}\right)$. From this we can conclude that introducing a third, uninformed bidder must change the picture: We find that $\pi_{20}(b) = \pi_{200}(b)$ and $\pi_{11}(b) < \pi_{110}(b)$ since having two uninformed bidders in the auction is not more profitable for the seller than having only one uninformed bidder. This implies $\pi_{200}\left(\frac{1}{6}\right) < \pi_{110}\left(\frac{1}{6}\right)$. Hence with a symmetric probability distribution F and more than two bidders, a fixed amount of information should rather be spread equally among two bidders than concentrated at one bidder. Thus there is no reason to expect the curves in Fig. 3 to coincide at $\frac{1}{6}$.

We close this discussion with the following observation: In contrast to the two bidder case and to our previous example, it can sometimes be strictly optimal to choose a symmetric allocation of packages when there are more than two bidders:

Example 4. Assume that the seller wants to allocate three packages of information among at least three bidders. If the packages are distributed uniformly on [0, 1] or exponentially with parameter 1, it is strictly most profitable for the seller to give the three packages to three different bidders.

5. Conclusion

We have studied an independent private values second-price auction with entry fees in which the seller can split up a fixed total amount of information differently among the bidders. We have found that if giving out information is costly, the seller often decides not to provide all information. Moreover, we saw that restricting the seller from allocating amounts of information symmetrically is a surprisingly strong restriction: In the two bidder case, choosing a symmetric allocation of information is essentially the worst decision the seller can take. Any other split-up of packages (at least into even numbers) would lead to higher revenues. Suboptimal allocations of information can lower the seller's revenues (and overall welfare) substantially.

Several examples in the paper have shown that the optimal information policy may depend sensitively on the probability distribution of the information packages. The best way for allocating information is thus a procedure that demands careful examination of the setting. To find a good information policy, there is no simple rule of thumb that can replace a thorough investigation of how bidders may incorporate additional information.

We want to point out that the difficulties we find in our simple model indicate that a more general analysis must be highly complex. For instance, we have seen that the optimal split-up of information depends very sensitively on how the bidders' valuations are distributed. We have seen as well that returns to giving out information are not monotonically decreasing. It remains a challenge to find a more tractable model that still allows for an easy and quite natural cardinal ranking of informativeness. All classes of models with release of information that contain our model of independent information packages will suffer from non-monotonicity and sensitivity with regard to distributional assumptions.

Appendix. Proofs

Proof of Proposition 1. Define $1_{\{i\}} = 1_{\{i \text{ wins}\}}$. To prove the formula for the entry fees, we only have to show that

$$E[(\widetilde{X}^{(1)} - \widetilde{X}^{(2)})1_{\{i\}}]$$

is the expected revenue of bidder i from the informed auction. Define $\overline{\mathcal{F}} = \sigma(\bigcup_i \mathcal{F}_i)$, i.e., $\overline{\mathcal{F}}$ is the smallest σ-algebra containing all $\mathcal{F}_i$. Then we have, using independence of the $\mathcal{F}_i$, $\overline{\mathcal{F}}$-measurability of $1_{\{i\}}$ and the law of iterated expectations:

$$\begin{aligned} E[\widetilde{X}^{(1)}1_{\{i\}}] &= E[E[X_i|\mathcal{F}_i]1_{\{i\}}] = E[E[X_i|\overline{\mathcal{F}}]1_{\{i\}}] \\ &= E[E[X_i 1_{\{i\}}|\overline{\mathcal{F}}]] = E[X_i 1_{\{i\}}]. \end{aligned}$$

We thus have

$$E[(\widetilde{X}^{(1)} - \widetilde{X}^{(2)})1_{\{i\}}] = E[(X_i - \widetilde{X}^{(2)})1_{\{i\}}] = e_i$$

which is the expected revenue of bidder i that the seller can extract as an entry fee. We further have to show that the seller's expected revenue is equal to

$$E[\widetilde{X}^{(1)}].$$

As $\sum_i 1_{\{i\}} = 1$, the agents' entry fees add up to

$$E[\widetilde{X}^{(1)} - \widetilde{X}^{(2)}].$$

Adding to this the expected selling price in the second-price auction, which is $E[\widetilde{X}^{(2)}]$, we are done. □

Proof of Proposition 2. The formula for the entry fees is proved by showing that the difference between the fees equals $|E[X_1] - E[X_2]|$ and their sum equals $E[|\widetilde{X}_1 - \widetilde{X}_2|]$. The two agents' entry fees as calculated in the previous proposition are

$$e_1 = E[(\widetilde{X}_1 - \widetilde{X}_2)1_{\{1 \text{ wins}\}}] \quad \text{and} \quad e_2 = E[(\widetilde{X}_2 - \widetilde{X}_1)1_{\{2 \text{ wins}\}}].$$

The difference in entry fees, $e_1 - e_2$, equals

$$\begin{aligned} &E[(\widetilde{X}_1 - \widetilde{X}_2)1_{\{1 \text{ wins}\}}] - E[(\widetilde{X}_2 - \widetilde{X}_1)1_{\{2 \text{ wins}\}}] \\ &\quad = E[(\widetilde{X}_1 - \widetilde{X}_2)1_{\{1 \text{ wins}\}}] + E[(\widetilde{X}_1 - \widetilde{X}_2)1_{\{2 \text{ wins}\}}] \\ &\quad = E[X_1] - E[X_2]. \end{aligned}$$

Their sum, $e_1 + e_2$, equals

$$\begin{aligned} &E[(\widetilde{X}_1 - \widetilde{X}_2)1_{\{1 \text{ wins}\}}] + E[(\widetilde{X}_2 - \widetilde{X}_1)1_{\{2 \text{ wins}\}}] \\ &\quad = E[(|\widetilde{X}_1 - \widetilde{X}_2|)1_{\{1 \text{ wins}\}}] + E[(|\widetilde{X}_1 - \widetilde{X}_2|)1_{\{2 \text{ wins}\}}] \\ &\quad = E[|\widetilde{X}_1 - \widetilde{X}_2|]. \quad \square \end{aligned}$$

Proof of Proposition 3. In this and the following proofs, we make use of the fact that for $k, j \leq m$ the seller's expected revenue from revealing $X_1, \ldots, X_k$ and $Y_1, \ldots, Y_j$ can be rewritten as

$$\begin{aligned} &E[\max(X_1 + \cdots + X_k + (m-k)\mu, Y_1 + \cdots + Y_j + (m-j)\mu)] \\ &\quad = m\mu + E[\max((X_1 - \mu) + \cdots + (X_k - \mu), (Y_1 - \mu) \\ &\qquad + \cdots + (Y_j - \mu))] \\ &\quad = m\mu + \frac{1}{2}E[|(X_1 - \mu) + \cdots + (X_k - \mu) - (Y_1 - \mu) \\ &\qquad - \cdots - (Y_j - \mu)|]. \end{aligned} \tag{5}$$

In order to compare the seller's expected revenue from different choices of k and j, we can concentrate on the second summand in the last expression. As this expression only depends on the random variables $X_i - \mu$ and $Y_i - \mu$, we can set $\mu = 0$ without loss of generality. To prove that giving all packages to one bidder weakly dominates the equal split-up, we just have to show that for independent, identically distributed, mean zero random variables X_i and Y_i

$$E[|X_1 + \cdots + X_k + X_{k+1} + \cdots + X_{2k}|] \geq E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_k|] \tag{6}$$

with equality exactly in the symmetric distribution case. Recall the inequality from Jagers et al. (1995) cited above: For X and Y i.i.d.

$$E[|X + Y|] \geq E[|X - Y|]$$

(with equality exactly in the symmetric case). Setting $X = X_1 + \cdots + X_k$ and $Y = Y_1 + \cdots + Y_k$, this becomes

$$E[|X_1 + \cdots + X_k + Y_1 + \cdots + Y_k|] \geq E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_k|].$$

By the i.i.d. assumption we can substitute $Y_1, \ldots, Y_k$ on the left hand side by $X_{k+1}, \ldots, X_{2k}$ and obtain (6). (Note that a sum of i.i.d. random variables is symmetric around its mean if and only if the summands are symmetric around their means.) □

Proof of Proposition 4. In order to circumvent an unnecessarily complicated notation for a simple variation of the proof of Proposition 3, we only show that six packages should rather be split up into four and two than into three and three. All the other inequalities covered by Proposition 4 follow from analogous arguments. By (5), we only have to show that for i.i.d. mean-zero random variables X_i and Y_i

$$E[|X_1 + X_2 + X_3 + X_4 - Y_1 - Y_2|] \geq E[|X_1 + X_2 + X_3 - Y_1 - Y_2 - Y_2|]. \tag{7}$$

We start again with

$$E[|X + Y|] \geq E[|X - Y|].$$

Setting $X = X_1 + X_2 - X_3$ and $Y = X_4 + X_5 - X_6$, this becomes

$$E[|X_1 + X_2 + X_4 + X_5 - X_3 - X_6|] \geq E[|X_1 + X_2 + X_6 - X_3 - X_4 - X_5|].$$

Using the i.i.d. assumption we can rename the summands on both sides of the inequality and obtain (7). To see that the distribution of $X_1 + X_2 - X_3$ is asymmetric if and only if the distribution of the X_i is asymmetric, note that $X_1 + X_2 - X_3$ is the sum of the (possibly asymmetric) random variable X_1 and the always symmetric random variable $X_2 - X_3$. The other inequalities covered by the proposition follow with parallel arguments, setting $X = X_1 + X_2 + X_3 - X_4$, then $X = X_1 + X_2 + X_3 - X_4 - X_5$, etc. □

Proof of Lemma 1. Consider $0 \leq k, j \leq m$ with $k + j \geq 2$ and $k \geq 1$. As adding another package always costs c, we only have to consider by how much a package raises the seller's expected gross revenue. It is sufficient to compare the increase in revenue from revealing the kth package to bidder 1 given that bidder 2 gets j packages with the revenue increase from revealing the first package to bidder 1 given that bidder 2 gets no package. So by (5), we have to show that for i.i.d. mean-zero random variables $X_1, \ldots, X_k$ and $Y_1, \ldots, Y_j$

$$E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_j| - |X_1 + \cdots + X_{k-1} - Y_1 - \cdots - Y_j|] < E[|X_1|].$$

The fact that this inequality holds weakly is an immediate consequence of the triangle inequality where we use that $E[|X_1|] = E[|X_k|]$. Since we have assumed the X_i and Y_i to be not a.s. constant, equality would contradict the independence of the X_i and Y_i. Thus we obtain a strict inequality and are done. □

Proof of Example 2. We only need to consider gross revenues since every additional package costs c. By (5), we know that, for the first two (symmetric) examples, it is sufficient to compare the increments of the sequence 0, $E[|X_1|]$, $E[|X_1 + X_2|]$, $E[|X_1 + X_2 + X_3|]$, $E[|X_1 + X_2 + X_3 + X_4|]$ for independent random variables X_i distributed according to the distributions from the examples but shifted to have mean zero. For the uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$, we obtain

$$E[|X_1|] = \frac{1}{4}, \qquad E[|X_1 + X_2|] = \frac{1}{3},$$
$$E[|X_1 + X_2 + X_3|] = \frac{13}{32}, \quad \text{and}$$
$$E[|X_1 + X_2 + X_3 + X_4|] = \frac{7}{15}.$$

For the distribution that takes $-\frac{1}{2}$ and $\frac{1}{2}$ with equal probability we get

$$E[|X_1|] = \frac{1}{2}, \qquad E[|X_1 + X_2|] = \frac{1}{2},$$
$$E[|X_1 + X_2 + X_3|] = \frac{3}{4}, \quad \text{and}$$
$$E[|X_1 + X_2 + X_3 + X_4|] = \frac{3}{4}.$$

For the third example where – because of the asymmetry – the order in which packages are allocated matters, we have to compare the increments of the sequence 0, $E[|X_1|]$, $E[|X_1+X_2|]$, $E[|X_1+X_2-X_3|]$, $E[|X_1+X_2-X_3-X_4|]$ to see how the different packages affect the seller's revenue. Here, the X_i are independent and distributed according to the exponential distribution shifted by its mean 1 to the left. We obtain

$$E[|X_1|] = 2e^{-1}, \qquad E[|X_1 + X_2|] = 8e^{-2},$$
$$E[|X_1 + X_2 - X_3|] = \frac{7}{2}e^{-1}, \quad \text{and}$$
$$E[|X_1 + X_2 - X_3 - X_4|] = \frac{3}{2}.$$

Calculating these expectations is tedious but straightforward. Besides the formulas for the distribution functions of sums of uniformly and exponentially distributed random variables from Feller (1971), the following result from Jagers et al. (1995) proved to be useful in the third example: Let X and Y be independent random variables with distribution functions F and G. Then it holds that

$$E[|X - Y|] = \int_{-\infty}^{\infty} F(x)(1 - G(x))dx + \int_{-\infty}^{\infty} G(x)(1 - F(x))dx. \quad \square$$

Proof of Lemma 2. Denote by X_i and Y_i the packages of information of the two agents normalized such that they have mean zero. By (5), we have to prove that the sequence of the seller's gross revenues

$$P_l := m\mu + \max_{0 \leq k \leq l} \frac{1}{2} E[|(X_1 - \mu) + \cdots + (X_k - \mu) - (Y_1 - \mu) - \cdots - (Y_{l-k} - \mu)|]$$

is bounded by $m\mu + \frac{\sigma}{2}\sqrt{l}$ and weakly increasing in l. Note that it is sufficient to prove this for $\mu = 0$. The upper bound on the sequence $(P_l)_l$ follows with Jensen's inequality:

$$E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k}|] = E[\sqrt{(X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k})^2}]$$

$$\leq \sqrt{\mathrm{Var}(X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k})}$$
$$= \sqrt{l\,\mathrm{Var}(X_1)} = \sqrt{l}\sigma.$$

To see that the sequence $(P_l)_l$ is weakly increasing we show that the optimal split-up of $l+1$ packages is at least as good as the optimal split-up of l packages. Choose a k with $0 \leq k \leq l$ which maximizes

$$E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k}|],$$

and set $A = X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k}$. Note that the function $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(z) \equiv E[|A + z|]$$

is convex. Thus, by Jensen's inequality,

$$g(0) = g(E[X_{k+1}]) \leq E[g(X_{k+1})].$$

Yet this is the same as

$$E[|X_1 + \cdots + X_k - Y_1 - \cdots - Y_{l-k}|]$$
$$\leq E[|X_1 + \cdots + X_{k+1} - Y_1 - \cdots - Y_{l-k}|].$$

Hence we have found a split-up of $l+1$ packages that leads to a weakly higher gross revenue than the optimal split-up of l packages. Thus also the optimal split-up of $l+1$ packages leads to a weakly higher gross revenue than the optimal split-up of l packages. □

Proof of Proposition 5. It is strictly more profitable to inform bidder 1 than to inform bidder 2, if

$$E[\max(X_1, \mu_Y)] > E[\max(\mu_X, Y_1)].$$

By the identity $\max(u, v) = \frac{1}{2}(u + v + |u - v|)$ this is equivalent to

$$E[|X_1 - \mu_Y|] > E[|Y_1 - \mu_X|]. \quad \square$$

Proof of Example 4. Denote the packages of the first three bidders by X_i, Y_i and Z_i, respectively. We have to show that for both distributions all the three packages should be given to three different bidders no matter whether the number of bidders is exactly 3 or greater. Thus we have to show that

$$E[\max(X_1 + X_2 + X_3, 3\mu)]$$
$$< E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu)]$$
$$E[\max(X_1 + X_2 + \mu, Y_1 + 2\mu, 3\mu)]$$
$$< E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu)]$$

for exactly three bidders and

$$E[\max(X_1 + X_2 + X_3, 3\mu)]$$
$$< E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu, 3\mu)]$$
$$E[\max(X_1 + X_2 + \mu, Y_1 + 2\mu, 3\mu)]$$
$$< E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu, 3\mu)]$$

for more than three bidders. Obviously, the first two of these inequalities imply the second two. The first two inequalities follow from the following calculations: For the exponential distribution (where $\mu = 1$), we have

$$E[\max(X_1 + X_2 + X_3, 3\mu)] = 3 + \frac{27}{2}e^{-3},$$

$$E[\max(X_1 + X_2 + \mu, Y_1 + 2\mu, 3\mu)] = 3 + e^{-1} + 4e^{-2} - \frac{7}{4}e^{-3},$$

$$E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu)] = \frac{23}{6}.$$

For the uniform distribution (where $\mu = \frac{1}{2}$), we have

$$E[\max(X_1 + X_2 + X_3, 3\mu)] = \frac{109}{64},$$

$$E[\max(X_1 + X_2 + \mu, Y_1 + 2\mu, 3\mu)] = \frac{671}{384},$$

$$E[\max(X_1 + 2\mu, Y_1 + 2\mu, Z_1 + 2\mu)] = \frac{7}{4}. \quad \square$$

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